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I. Introduction

We propose that baroclinic instability plays an important role in the generation and stability of the strong zonal jets observed on the outer planets of the solar system. In the past, two general approaches have been taken to explain these jets. Busse (1976), inspired by the Taylor-Proudman effect, suggested that if the flow is deep and extends all the way through the planet, then the jets may be the surface manifestation of differentially rotating cylinders concentric with the planet's rotating axis. From a very different perspective, Rhines (1975), assuming the dynamics are confined to an outer "weather layer", suggested that the zonal jets emerge from decaying turbulence on a beta plane. Here, we suggest a new approach which contains some elements of the two, and in addition suggests an energy source for the jets and gives a dynamical explanation for the observed multi-jet structure and its stability.

III. Unstable Modes

Observations indicate that the gas planets of the solar system have a very weak thermal equator to pole temperature gradient. Therefore thermal wind balance implies a very weak global baroclinic shear. Standard baroclinic instability theory requires a minimal vertical shear for instability. However, since here the planetary potential vorticity has opposite signs in the two layers, instability can occur with very little baroclinic shear. Moreover, this instability appears at high wave numbers (Fig. 2) which as we show favors rapid meridional variation in the induced zonal flow.

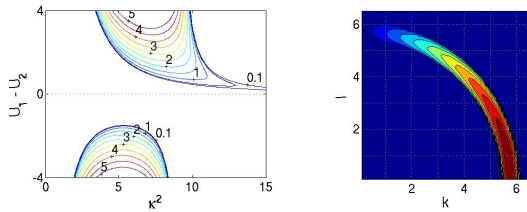


Figure 2: Linear stability calculation; Growth rate in wave number space (right); growth rate in shear - total wave number space (left), redder shades are bigger growth rates.

V. Generation of Jets

We perform numerical experiments with a full nonlinear pseudo-spectral model containing a full spectrum of initial perturbation modes. By setting different basic shear levels, different initial modes become unstable (Fig. 2). We find that the mechanism seen analytically in the truncated model of the formation of a highly oscillatory meridional zonal wind structure, is seen also in the full model and leads to the formation of jets. An example of the evolution leading to the formation of jets is presented in Fig. 4. Initially a small random perturbation is applied to the basic state flow (4a). At some time the fastest growing mode (in this case $k=7, l=5$) becomes dominant (4b) and the perturbation grows baroclinically by an order of magnitude. Then, as predicted by the truncated model, the nonlinear interactions form an induced zonal flow with several jets (4c). In time, more modes come in; the flow becomes turbulent, and a quasi-geostrophic inverse energy cascade begins setting three major jets (4d) in the channel (one easterly and two westerly) with a typical width on the order of the Rhines scale.

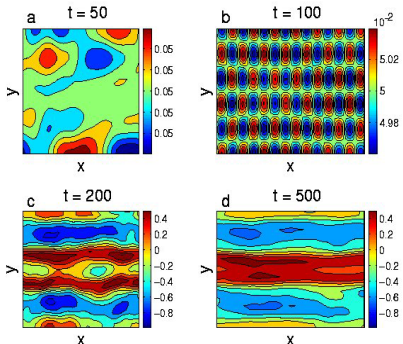


Figure 4: The evolution of the zonal velocity from a random perturbation to a zonal jet.

These jets now are very stable in time. Unlike previous models published, these jets appear in the instantaneous pictures and not only in the zonally averaged ones. The total zonal velocity is composed of three components; the first being the basic flow which is constant creating the vertical shear and is invariant under translation between the two layers; the second being the induced zonal flow created from the self interaction of the eddy field and the third being the eddy field itself.

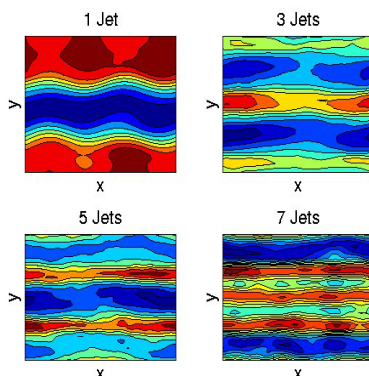


Figure 5: Snapshots of the end state total zonal velocity field, for cases of different basic initial shears.

VIII. Summary

- We propose a mechanism for the generation and stability of atmospheric jets in gas planets with a deep convective atmosphere.
- In the existence of a convective interior, baroclinic instability can exist even for weak vertical wind shears.
- Baroclinic instability can remove energy from the mean shear and transform it to eddies. Self interaction of the eddies causes an induced zonal flow with a multi-jet meridional structure which can be as strong as the other components of the zonal velocity.
- Self interaction of the eddies halts the instability and stabilizes the flow via exchange of energy between the eddies and the mean flow.
- The truncated model containing only the fastest growing mode predicts well the initial instability and the structure of the induced zonal flow.

II. The Model

We use a two-layer quasi-geostrophic model (Phillips, 1951). Unlike the standard two layer model, here the lower layer is taken to be much deeper than the upper layer, and the lower layer is parameterized using a negative β plane to represent a convective column structure penetrating through the planet, without including the details of convection. Illustratively, on a spherical shell a fluid column which is shifted towards the axis of rotation tends to shrink, while a fluid column which penetrates through the inside of a sphere (Fig. 1) will stretch when shifted towards the axis of rotation. This opposite effect suggests that such an inner geometry is equivalent to a negative β plane effect for the lower layer. More rigorously, Ingersoll and Pollard (1982) showed that when considering the weak deviations from motions constricted to a convective cylindrical annuli, the equivalent barotropic stability has an effective β which is negative. The interface between the two layers is a free surface, each layer is a homogeneous fluid with constant density and both layers are confined meridionally to a channel and rotate at a constant rotation rate.

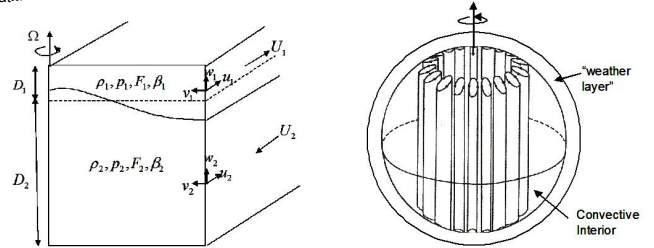
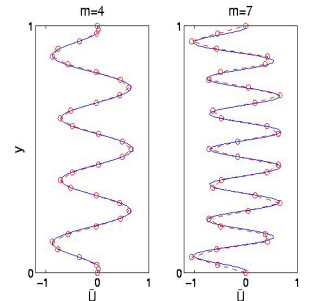


Figure 1: Scheme of two layer model; Spherical inner convective layer and outer standard atmospheric layer (right); Cartesian two layer - two β model (left).

IV. Truncated Model

Truncating the perturbation to one unstable mode, the full nonlinear quasi-geostrophic potential vorticity equation can be solved directly to give the induced zonal baroclinic correction to the mean flow due to the nonlinear interactions of the eddies. This velocity is given by

$$U_c(y, t) = \frac{2\pi m q'_c(t)}{4\pi^2 m^2 + F_1 + F_2} \cdot \left[\cos(2\pi m y) - \frac{\cosh\left(\sqrt{F_1 + F_2}\left(y - \frac{1}{2}\right)\right)}{\cosh\left(\frac{\sqrt{F_1 + F_2}}{2}\right)} \right]$$



It gives an oscillation in the meridional structure of the induced zonal flow. Fig. 3 shows this structure for specific modes (blue) on top of a solution from the full nonlinear model containing all modes (red). The full model is dominated by the fastest growing mode and therefore matches well the truncated prediction.

Figure 3: The induced baroclinic zonal velocity U_c meridional profile for modes $m=4,7$.

VI. The Baroclinic Induced Zonal Flow

The truncated model predicts the formation of the induced zonal flow from the nonlinear interactions of the eddy field. Fig. 6 shows development of the induced zonal flow in time from the full model. It begins from a weak random field (a) until the fastest growing mode picks up. As this mode grows (b), an induced meridionally varying zonal flow emerges matching the prediction of the truncated model (superimposed by the dashed line), until the nonlinearities become big enough that more modes come in. Then the flow becomes turbulent (c); some jets converge (d) and the jets scales increase in agreement with quasi-geostrophic turbulence theory (Rhines, 1975) to the Rhines scale (shown by the thick solid line). Once it reaches the Rhines scale the induced zonal flow remains stable (e).

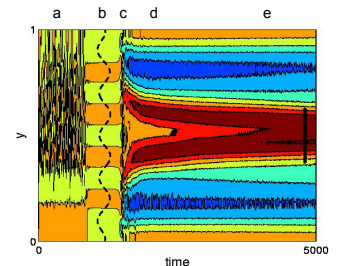


Figure 6: The evolution of the induced zonal velocity U_c .

VII. Nonlinear Equilibration

The stability of the flow is caused by the nonlinear interaction of eddies. When the flow is unstable the eddies grow and take energy from the from the basic state shear. Eventually the eddies have taken enough energy out of the basic state to change it so that the flow may become stable (cf. Pedlosky, 1970). In a truncated model containing only one mode this exchange of energy between the basic state and the perturbation gives an oscillation in the perturbation entrophy (Fig. 7), while in a model with many modes this leads to leveling of the entrophy. Fig. 7 shows how initially when the eddies are weak the truncated and the full model follow the linear growth curve until they both diverge from this curve due to the growing nonlinear interactions of the eddies. These nonlinear interactions cause the induced correction to the basic flow, which affects the basic shear, brings in other modes, and thus creates such a cycle which leads to the leveling of the entrophy due to oscillations of the entrophy in many modes.

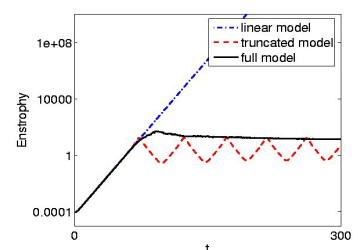


Figure 7: The perturbation entrophy as a function of time in the different models.

IX. References

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