Equatorial Super-rotation on Gas Giants Driven by Internal Convection

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I. Abstract

Jupiter and Saturn's atmospheres are dominated by multiple zonal jets with strong superrotation around the equator. We have modified the MIT general circulation model to be suitable for giant planets. The model's geometry is a full 3D sphere down to an inner core (unlike the traditional spherical shell). It is non-hydrostatic, anelastic, uses an equation of state suitable for hydrogen-helium mixtures (SCVH), and is driven by internal heat. In the parameter regime suitable for giant planets the convective plumes tend to align with the direction of the rotation axis. We find that the baroclinic density gradients caused by the convection drive the system away from the Taylor-Proudman constraint, to a state closer to having the total zonal momentum rather than the velocity itself constant along the rotation axis. The convection drives large positive vorticity columnar eddies supporting superrotating flows around the equator.

II. The Model

We have modified the MIT general circulation model (MITgcm), augmenting the non-hydrostatic version so that the grid can reach deep into the planet's interior (including the strong variations in gravity). This extension basically allows the GCM to treat the complete dynamics of a whole sphere of gas instead of just a spherical shell. To account for the vertical variation in mean density we use the anelastic approximation. We use an equation of state for hydrogen and helium mixtures (Saumon et al. 1995) which accounts for some of the complex interior thermodynamical processes. In summary the main features of the model are:

- **Non-Hydrostatic**
- **Deep spherical geometry**
- **Non-Boussinesq - Anelastic**
- **Varying gravitational acceleration**
- **Hydrogen-Helium equation of state**
- **Forcing by interior and solar heat fluxes**

Model Equations

$$\bar{\rho}\frac{D\vec{u}}{Dt}+2\Omega\times\bar{\rho}\vec{u}=-\nabla P'-\rho' g(r)\hat{r}+\bar{\rho}\nu\nabla^2\vec{u}$$

$$\nabla\cdot\left(\bar{\rho}\vec{u}\right)=0 \qquad \rho=\bar{\rho}(r)+\rho'\left(s,r,t\right)$$

$$\frac{Ds'}{Dt}=\frac{\bar{S}Q}{\bar{T}}$$

The interior forcing is set by a vertical heating profile (simulating the cooling of the planet) with a magnitude set by the three control parameters which define the system: The Prandtl, Rayleigh and Taylor numbers

$$Pr=\frac{\nu}{\kappa} \qquad Ta=\frac{\Omega^2 H^4}{\nu^2} \qquad Ra=\frac{B_0 H^4}{\nu\kappa}$$

Model Thermodynamics

The gas is primarily composed of hydrogen and helium with small amounts of heavier elements. At low temperatures and pressures in the outer regions of the planet, hydrogen is a molecular gas and the EOS may be approximated as an ideal gas. Deeper into the interior, however, due to the high densities and relatively low temperatures (compared to stars), the giant planets lie in an extremely complex thermodynamic region. The main factors that separate the gas under these conditions from ideal gas behavior are pressure ionization, electron degeneracy, and Coulomb interactions. We use the SCVH equation of state (Saumon et al., 1995), calculated specifically for high pressure hydrogen and helium mixtures, including these thermodynamic complexities. The pressure-temperature-density relationship is shown in Fig.1, where it can be seen that up to about 1MBar (~0.9 the radius of the planet) the SCVH EOS is close to an ideal gas, but it differs substantially for the deep interior.

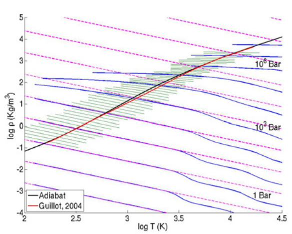

Figure 1: The SCVH eos pressure contours (blue), ideal gas pressure contours (pink) and the adiabatic reference state (black). Model layers are in green.

For the dynamical equations to remain energetically consistent when incorporating the anelastic approximation the system must have an adiabatic reference state. We set the reference state to follow the adiabat that matches the Galileo probe measurements (Atkinson et al., 1996). The variation from this reference entropy is computed dynamically.

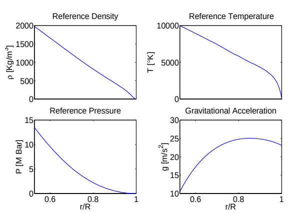

Figure 2: The adiabatic reference state of the model

For resolving the convection the layer depths follow a constant mean pressure gradient where the depth of each layer is then set following calculations of Guillot & Morel (1995). This allows to calculate the reference temperature, density and gravitational acceleration (Fig 2). For every layer separately we then fit a polynomial to the SCVH EOS for the variation in density so that

$$\rho'(p,s)=\left(\frac{\partial\rho}{\partial s}\right)_p s'+\left(\frac{\partial\rho}{\partial p}\right)_s p'$$

The derivatives are calculated from the SCVH polynomial for every reference pressure, and the primed variables are the anomaly at every grid cell. This variation in density feeds back to the model dynamics, coupling the dynamics and thermodynamics.

III. Basic Balances & 3D Fields

In the parameter regime suitable for Jupiter and Saturn the system in steady state is to first order geostrophic and hydrostatic. Since the aspect ratio is not small this includes all four Coriolis terms, and the numerical results show that the vertical Coriolis contribution is significant in the balance between pressure and buoyancy. The basic steady state balance is therefore

$$2\Omega\times\bar{\rho}\vec{u}=-\nabla P'-\rho' g(r)\hat{r}$$

This implies that thermal wind balance on the sphere will have the form

$$\frac{\partial u}{\partial r}\sin\theta+\frac{\partial u}{\partial\theta}\cos\theta=\frac{g}{2\Omega\bar{\rho}}\frac{1}{r}\frac{\partial\rho'}{\partial\theta}-\frac{u}{\bar{\rho}r}\frac{\partial\rho'}{\partial\theta}-\frac{1}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r}u\sin\theta=\frac{\partial u}{\partial"z"}$$

The 3D fields are presented with three slices through the sphere. In Fig. 3 the top panels are slices through the meridional plane (radius-latitude), the middle three are in the equatorial plane (radius – longitude), and the bottom three are on the surface (latitude – longitude) of a 90° sector in longitude. The left panels are the zonal velocity field, the middle ones are the potential temperature anomaly and the right ones are the 2D sreamfunction (vorticity in surface plot). Since the Rossby number is small and the system is convective the motion tends to align with the direction of the axis of rotation (see V).

Due to the mean density gradient (see IV) the zonal velocities are stronger near the upper levels. The zonal velocities have strong supperrotation at low latitudes, and a counter weak retrograde flow in the interior. Large positive vorticity columnar eddies are embedded in this shear (see VI).

The potential temperature anomaly is higher at the poles due to the alignment of the convective plumes with rotation axis. Analysis of the heat budget shows that the poleward transfer of heat is mainly due to the eddy heat fluxes. This mechanism transfers heat poleward which then may balance the solar heating resulting in a weaker atmospheric meridional temperature gradient.

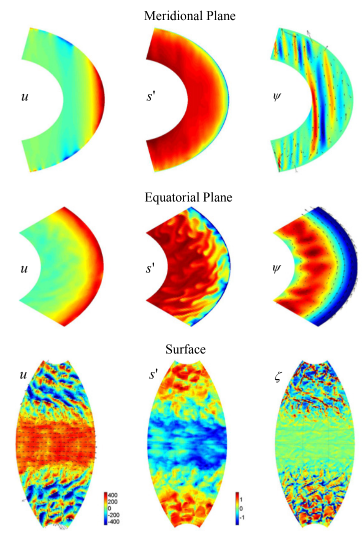

Figure 3: Meridional, equatorial and surface slices showing the zonal velocity (*m/s*), anomaly potential temperature (*K*), and 2D streamfunction (vorticity for surface plot). The run parameters are *Ra=5E7, Pr=10* and *Ta=2E8* (*Ek=1.5E-4*). Colorbars are the same.

IV. The Effect of the Mean Density Gradient: Anelastic vs. Boussinesq

One of the key questions on the giant planet's atmospheres is what is the baroclinic structure of the zonal winds. To date most models (c.f. Heimpel et al., 1995) used the Boussinesq approximation, but clearly the mean density variation plays an important role in setting this vertical velocity structure. Here we use an anelastic model to account for this effect. In Fig. 4 we compare a Boussinesq to an anelastic run, where all parameters are similar only that in the Boussinesq run the mean density is set constant. For small Rossby numbers and using the anelastic approximation the vorticity equation can be written as:

$$2\Omega\cdot\nabla\left(\bar{\rho}\vec{u}\right)=\frac{1}{\rho}\nabla\bar{\rho}\times\nabla p-\frac{1}{\rho}\nabla\rho\times\nabla p=\nabla\rho'\times g=\alpha\nabla s'+\beta\nabla p'$$

Note the importance of having the density anomaly depend on the pressure variation (section II) since the pressure term balances the mean density gradient on the left hand side. In the barotropic limit the anelastic form of the vorticity equation reduces to the standard form with the zonal velocity constant along the axis of rotation. However if the baroclinic contribution is not negligible then the Boussinesq and anelastic forms differ. We find that in both models the scales of the convectively driven density gradients do not vary in depth, and therefore in the Boussinesq model we still find close to Taylor columns. In the anelastic model however, due to the variation of the mean density, the vertical shear in the direction along the axis of rotation scales inversely with the mean density.

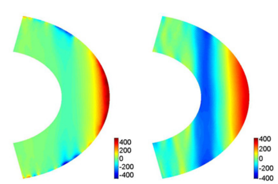

Figure 4: Comparison of the zonal velocity (*m/s*) structure between an anelastic and Boussinesq run. In both runs the parameters are *Ra=5E7, Pr=10,* and *Ta=2E8.*

V. Effect of Rotation

The system is convective and therefore plumes are driven by buoyancy away from the center of gravity. However since the Rossby number is small the convective motion tends to align with the direction of the axis of rotation. The ratio between the magnitude of the buoyancy frequency and the rotation frequency sets how aligned the plumes will be with the rotation axis. In Fig. 5 we show two identical experiments which vary only in the rotation frequency. The fast rotator has convective motion aligned with the rotation axis while the slow one has a more homogenous distribution of plumes. Both runs are well into the turbulent regime (Rayleigh number is ~100 times critical), and after the initial spin up have reached a statistical steady state with sporadic emergence of plumes.

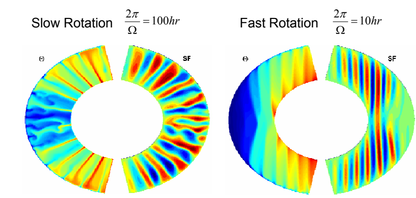

Figure 5: Two runs with all parameters set the same except the rotation period. The right panel in each is the 2D stream function, and the left is the potential temperature anomaly. The high Taylor number run is dominated by motion parallel to the rotation axis. These plots are from axisymmetric runs.

VI. Anelastic Momentum Columns & Super-rotating Equatorial Flow

Large scale eddies form on the equatorial plane. All eddies have positive vorticity (in respect to the direction on the rotation axis) and are embedded in the mean vertical shear flow. In Fig. 6 we plot slices in the radius – longitude plane (the plane on a certain latitude - from the center of the planet outwards). The planes in the plot are spaced by 10° in latitude. The higher the latitude the further away from the center of the planet the large eddies are, such that the eddies are all aligned parallel to the axis of rotation. These are columnar structures resulting from the rotation, but unlike Taylor columns they are in a state close to having the momentum constant along the axis of rotation rather than the velocity itself.

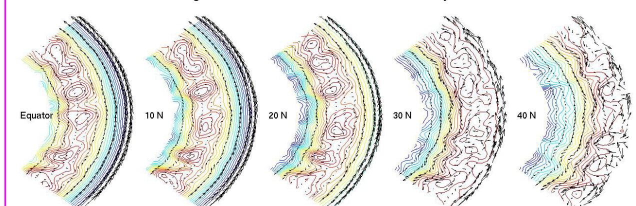

Figure 6: 2D streamfunction on the radius – longitude slices along constant latitude, spaced by 10° in latitude. The run parameters are *Ra=5E7, Pr=10* and *Ta=2E8*

The leading order zonal momentum balance is between the radial eddy fluxes and the radial viscous fluxes.

$$\frac{\partial}{\partial r}\left(\overline{w'u'}\right)\cong\nu\frac{\partial^2\bar{u}}{\partial r^2}$$

Given the boundary conditions this gives a correlation between $\overline{w'u'}$ and the mean zonal shear. Thus the positive shear is supported by positive vorticity transported along the momentum columns.

VII. Summary

- In a rapidly rotating system the character of convective turbulence is strongly affected by the ratio of the magnitude of the buoyancy frequency to the rotation frequency. (see part V)
- In an anelastic system columns develop along the direction parallel to the axis of rotation, but unlike Taylor columns they are closer to having the zonal momentum constant along the axis of rotation rather than the velocity itself . (see part IV)
- Eddy heat fluxes transfer heat parallel to the rotation axis resulting in heating the high lat. more than the low lat. and by that reducing the meridional temperature gradient caused by solar heating. (see part III)
- The convectively driven large columnar eddies have positive vorticity resulting in supperrotation around the equator, with the radial eddy fluxes balancing the mean shear. (see part VI)

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