ACM/ESE 118

An example of principal component analysis

In electrophysiological recordings of neural activity, one electrode typically records the action potentials (spikes) of several neurons. Before one can use the recorded spikes, for example, to study the coding of information in the brain, it is necessary to associate the spikes recorded with one electrode with one or more neurons, a procedure called spike sorting. Spikes can be associated with individual neurons because the recorded spikes of each neuron often have characteristic shapes. For example, Fig. 1 shows two different shapes of action potentials recorded with one electrode (data courtesy of Rajan Bhattacharyya). The two spikes are probably due

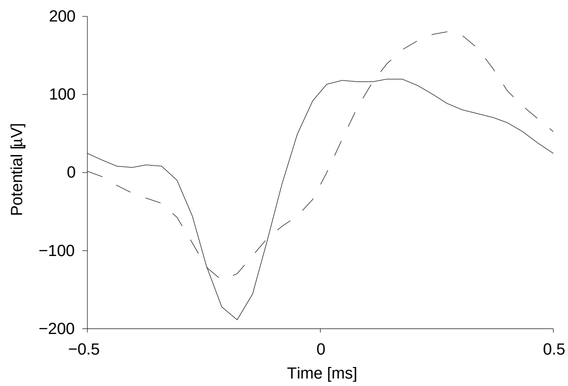

FIGURE 1: Two shapes of action potentials.

to two different neurons.

Principal component analysis can be used to identify the principal variations of spike shapes. In this example, the $n \times p$ data matrix X contains $n = 16977$ recorded spikes, sampled at $p = 32$ points in time. (The time axis in Fig. 1 consists of 32 discrete points.) Using Matlab, we can center the data matrix by subtracting the mean of the columns from each of the n rows of the matrix:

```
Xm = mean(X);
Xc = X - repmat(Xm, n, 1);
```

The mean of the columns `Xm` is a mean spike, an average over the different spikes recorded with the electrode (Fig. 2). Principal component analysis will reveal those variations of spike shape about this mean that have largest variance. To the extent that variations of spike shape among neurons are larger than variations of spike

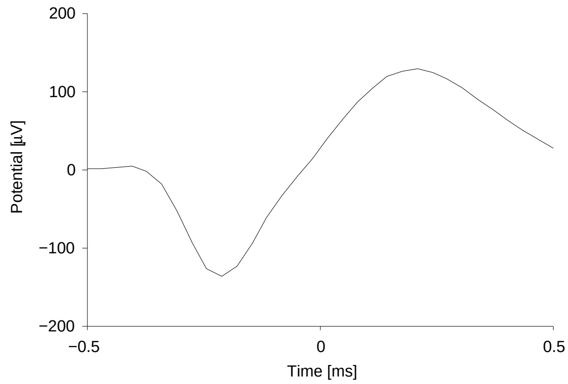

FIGURE 2: Mean shape of recorded action potentials.

shape for a single neuron, the principal component analysis can help to associate spikes with individual neurons.

The Matlab function `princomp` (or R function `princomp`) computes the principal component analysis via a singular value decomposition of the data matrix `X`:

```
[V, pcscores, pcvar] = princomp(Xc);
```

A plot of the spectrum of standard deviations (`sqrt(pcvar)`) of the principal components (Fig. 3) shows a clear spectral gap between the second and third principal component. The standard deviation of the third principal component is about

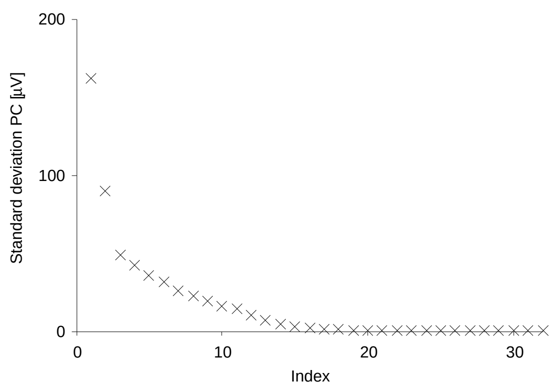

FIGURE 3: Standard deviations of principal components.

50μV, which is of the order of the standard deviation of the noise of the recordings. It therefore appears that the first two principal components, which together account for 80% of the total sample variation, contain most of the information about the spike signals (although a few higher-order principal components might still contain useful information).

Figure 4 shows the first two principal component vectors. Normalized to norm one, the principal component vectors represent the principal variations of spike shapes relative to the mean shape shown in Fig. 2.

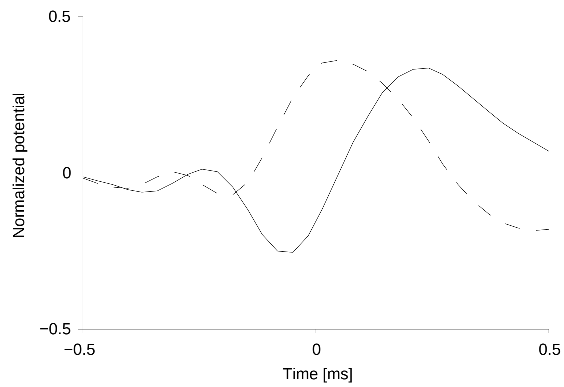

FIGURE 4: First principal component vector (solid) and second principal component vector (dashed) of variations of spike shape.

A scatterplot of the first two principal component scores (`pcscores = Xc*V`) reveals that the variations in spike shapes represented by the first two principal components form three clusters (Fig. 5). The three clusters may be identified with

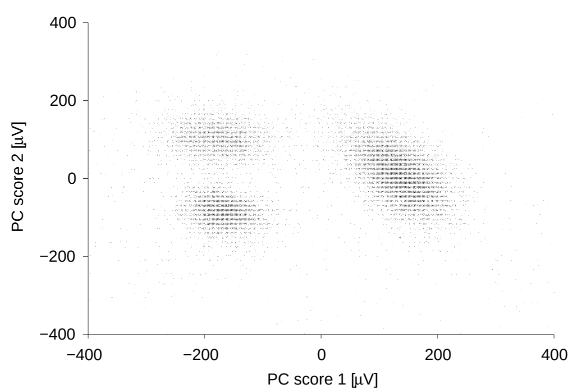

FIGURE 5: Scatterplot of the first two principal component scores.

three individual neurons. In this example, the mean spike shape (Fig. 2) does not seem to be a characteristic spike shape of any of these three neurons: the point $(0, 0)$ in the plane of the first two principal component scores does not characterize any of the three clusters. One cluster might be characterized by a score of $150\mu\text{V}$ of the first principal component and a score of zero of the second principal component. The other two clusters might be characterized by a score of $-170\mu\text{V}$ of the first principal component and a score $\pm 100\mu\text{V}$ of the second principal component. The corresponding spike shapes

```
shape1 = Xm + 150*V(:,1)';
shape2 = Xm - 170*V(:,1)' + 100*V(:,2)';
shape3 = Xm - 170*V(:,1)' - 100*V(:,2)';
```

are depicted as the solid, dashed, and dash-dotted lines in Fig. 6.

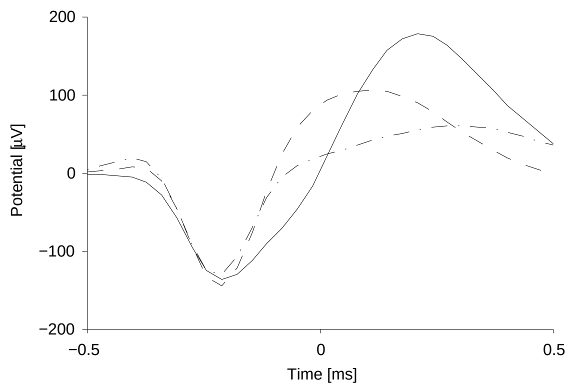

FIGURE 6: Characteristic shapes of action potentials corresponding to the three clusters in Fig. 5.