

Ge 131 2001

Homework # 5

Made Available, Friday November 2

Due Friday, November 9, 5pm

1. In the stagnant lid regime, the convective heat flow scales as  $\nu^{-1/3}$ , where  $\nu$  is the mantle viscosity. Everything else in the formula for heat flow is constant or nearly so, but the viscosity is very strongly temperature dependent.

(a) Assuming that this heat flow is balanced by instantaneous radioactive heating (i.e. neglecting for the moment the cooling term in the first law of thermodynamics applied to a planet), show that

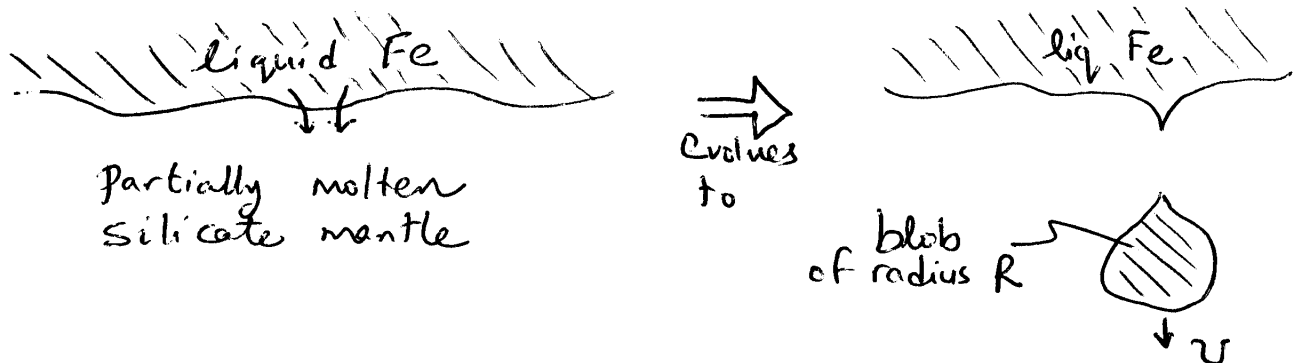
$$\frac{dT}{dt} = -\frac{3\lambda}{\gamma}; \quad \lambda = -\frac{d \ln Q}{dt}, \quad \gamma = -\frac{d \ln \nu}{dT}$$

where  $Q(t)$  is the radioactive heating and  $T$  is the mantle temperature. Don't assume  $\lambda$  is constant!

(b) What does this result predict for the *change* of cooling rate (i.e.,  $-d^2T/dt^2$ )? I'm not expecting a numerical answer but rather an estimate of the factor by which the cooling rate ( $-dT/dt$ ) will have changed over 4.5Ga. [Hint: think about the sources for  $Q$  and their respective decay times. In other words, how does  $\lambda$  vary with time?]

(c) Explain how this could lead to core convection turning off. [Hint: The core will try to cool at the same rate as the mantle. So mantle cooling rate may tell you core heat flow, at least for so long as the core convects.]

2. In models for core formation in terrestrial planets, it is often envisaged that a layer of liquid iron becomes Rayleigh-Taylor unstable and settles towards the center.



(a) Confirm that the growth rate we calculated in class for this instability is consistent with Stokes formula for a sinking blob ( $v = 2g\Delta\rho R^2/9\eta$ ; where  $\Delta\rho$  is

the density difference between iron and neighboring mantle,  $R$  is the blob radius and  $\eta$  is the dynamic viscosity. "Consistent with" means that the instability growth rate times wavelength equals velocity. Why? [Note: You will not, of course, get the factor of  $2/9$ .]

(b) It is believed on the basis of geochemical studies (the partitioning of siderophiles between the iron and neighboring mantle) that these core-forming blobs did not reequilibrate with the mantle through which they descended. This means that the volume of mantle that all the blobs cumulatively equilibrated with by diffusion is small compared with the total mantle volume. Let  $D$  be the relevant diffusivity. Explain why a blob of radius  $R$  can be expected to equilibrate with a volume of mantle material of order  $2\pi R \cdot (DR/v)^{1/2} \cdot H$  where  $v$  is the Stokes fall velocity and  $H$  is the total descent distance. (The way I've written this formula is an enormous hint as to where it comes from. I'm assuming  $R \gg (DR/v)^{1/2}$ ).

(c) Hence derive an estimate for the smallest blobs that lead to core formation, assuming Earthlike parameters for  $g$ ,  $H$ , etc.; and  $\eta = 10^{20}$  Poise,  $D = 10^{-5}$  cm<sup>2</sup>/sec (corresponding to partially molten mantle). What does this predict for the "longest possible" core formation timescale (here defined to be  $H/v$  with  $R$  given by the smallest acceptable blob size)? Assume all blobs have the same size. Of course, core formation might in reality take longer because the blobs start at different times.