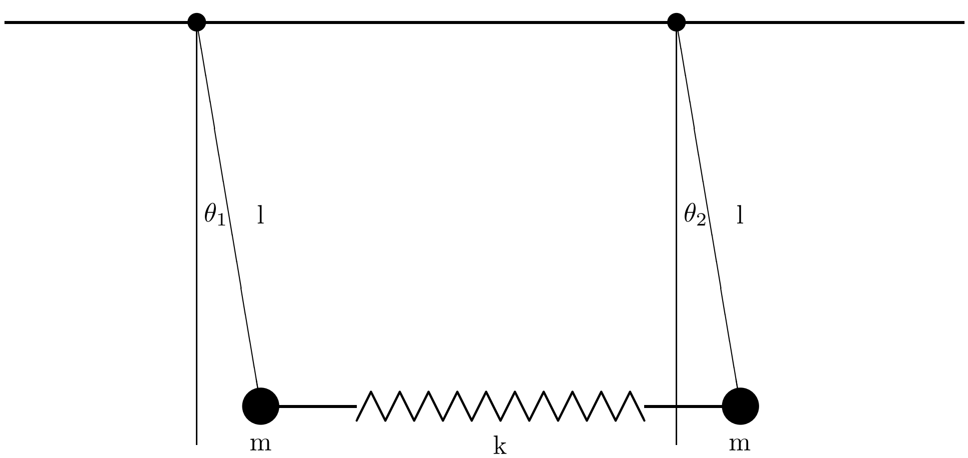
Problem Set # 7

Ge 108

Due 23 November 1998

1 Double your pleasure

Consider a system, as illustrated below, that has two pendula of length l, with a mass m at the bottom of each, that are connected by a spring with spring constant k.



(a) What are the equations of motion for the two masses, assuming that the pendulum motions are small (so $\theta = \sin\theta$) and that the spring can be approximated to be completely horizonal at all times (this would be impossible, of course, but what we are really assuming is that the vertical motions of the springs are much much smaller than the length of the spring, so that we need only consider the horizonal motion of the masses for calculating the stretching of the spring).

(b) What are the eigenfrequencies of this oscillation?
(c) What are the eigenmodes?

2 Bondage

HCN and HNC are very different molecules, because inside them different atoms attach to each other. Approximate the bonds as springs, as in the figure below, and determine and compare the vibration modes of the two molecules.

3 L.A. Strings

For an infinite string with mass per unit length p and tension T, we found the wave equation to be
$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$
and we found that one solution to the equation was
$$u(x,t) = u_1 \exp(ikx + ikvt) + u_2 \exp(ikx - ikvt).$$
Show that, in fact, *any* function of the form
$$u(x,t) = f(x + vt) + g(x - vt)$$
is a solution to the wave equation.

Consider now a function $h(x)$ where, for all values of x less than zero or greater than one, the function is equal to zero. Between 0 and 1, though $h(x)$ is a silouhette of the downtown LA skyline (hmmm... is there such a thing?).

If $u(0) = h(x)$ and $\dot{u}(0) = v\frac{dh}{dx}$, what does the solution to the wave equation look like at a later time? Sketch the solution at a few times in the future. What if $\dot{u}(0) = -v\frac{dh}{dx}$? If $\dot{u}(0) = 0$?

4 Numerology

Track down the program `waves.pro` from the web page. This program solves the one-dimensional wave equation given any arbitrary initial conditions and boundry conditions (although it is set up to have a wall at each outer boundry). In our numerics lecture, we didn't talk specifically about how to solve the wave equation. It is sufficiently different from the diffusion equation that a simple straight forward differencing actually doesn't work very well (the reason is that with the diffusion equation everything is diffusing its way towards zero, so small errors diffuse away, too, while for the wave equation structures remain large and move around, so errors stay large and grow). To make things more accurate, we really should implement an *explicit* integration scheme, where the values of the time derivative are taken from the spatial derivative *at the same time* instead of at the previous timestep (we *did* discuss this technique a bit in the numerics lecture of diffusion). All of this explanation is only an excuse for then saying that we will do it the simple yet significantly less accurate way. If ever you wanted to solve a wave equation type thing in real life, you would want to look up in *Numeric Recipes* how to do it correctly. Nonetheless, here we go.

(a) Make initial conditions such that the string position is zero everywhere except for between positions 40 and 60, where the string is initially a triangular function which rises from 0 at 40 to 1 at 50 and falls back to 0 at 60. What is the anlytic solution to the wave equation? How does it compare to the previous problem? What happens when the wave hits the wall?

(b) Add a forcing term such that the left-hand side of the string is pulled up and down with a cosine motion with an amplitude of 0.1. What if the frequency of the motion is equal to the fundamental frequency of vibration of the string? What if it is equal to the first overtone? What if it is half of the fundamental frequency? Note that in all of these examples there is no dissipation (damping) and we are continuoously adding energy, so the motions should continuously grow. How would you add a damping term to the string?