Final

Ge 108

Due 5pm, 9 December 2011, in Adam's mailbox

This exam has a 6-hour time limit. You may use notes you took in class, notes handed out in class, problem sets, problem set solutions, and computers calculators or abacuses, but no other outside sources. You must work independently. In several cases, you might need standard constants, such as the radius of the earth, the density of water, or physical constants. You can look these up anywhere you please.

1 What a drag

In a very thin atmosphere (like the earth's above 150 km) the atmospheric drag on a projectile is proportional to the first power of the velocity through the atmosphere (just like damping in an oscillator), with a proportionality constant of γ.

a) Assuming there are no other outside forces on the object (not even gravity!), what is the second-order differential equation of motion for the object?

b) This second order equation can be re-written as two first-order equations. One of the equations is obvious and looks something like $v = dx/dt$. What is the other one?

c) Solve these equations for the case of initial velocity v_0 and initial position x_0 to get a function for position as a function of time t.

d) Qualitatively describe the motion of the particle.

2 Spring thing

A set of masses coupled to springs is set up just like in Figure 13.1 (pg. 106) with all spring constants equal to k and both masses equal to m.

a) Write full solution for the motion of both masses for the initial conditions that $dx_1/dt = dx_2/dt = 0$, x_1=2C, and $x_2 = -C$, where C is a constant and the positive and negative values of the xs are defined as in Figure 13.1. (Note that there is an easy way and a hard way to do this problem.)

b) Assume that the springs are now damped with damping constant γ and that they are underdamped. Qualitatively describe the behavior of the motion as a function of time by considering the damping time scales of the different modes of oscillatation.

c) Using the assumptions of (b), mass 1 is now forced with an amplitude B and a frequency ω. Qualitatively sketch the amplitude of oscillation of mass 2 as a function of ω, being sure to carefully consider the behavior at ω of zero and infinity.

3 Silly strings

a) A horizonal string of length l is attached to a vertical pole with a ring that can slide up and down the pole. The pole is, of course, frictionless. The other end of the string is attached to a wall. Using no math, draw the first couple of fundamental oscillation modes of the string. Think carefully about what happens at the ring and how that differs from what happens at a wall.

b) Mathematically, what are the boundary conditions that this string must satisfy? (It is easiest to simply intuit these, but if you prefer the harder route, you should use the fact that the only force acting on the infinitessimally small piece of string at the end is the force caused by the displacement of that piece of string from the piece of string immediately adjacent to it. In the limit when infinitessimal goes to zero, you get a boundary solution for the spatial derivative of the spatial function.)

c) Solve the wave equation with these boundary conditions. What are the frequencies and shapes of the first few fundamental modes?

d) How do these modes relate to those on a string of length $2l$ which is attached to a wall at both ends (no rings!)?

4 Meaningless math

a) We have solved most simple yet physically meaningful PDEs in class already, so let's make up one that is not physically meaningful.

$$\frac{\partial u}{\partial t} = R\frac{\partial u}{\partial x}. \tag{1}$$

As far as I know, this equation describes no real system (though bonus points if you can prove otherwise!). But let's solve it anyway. As a partial differential equation, there is no simple integral we can do to get a solution. But, as always, we can search for classes of solutions where the spatial and temporal parts of the solution are separable. Find such a class of solutions.

b) Using this general solution, find a specific solution that neither decays away with time nor becomes infinite with time.

c) If the initial conditions are

$$u(x, t = 0) = A\cos(\lambda x)$$

write the full time variable solution to the equation.

d) With what velocity or velocities does the solution travel and in which direction or directions?