

## GE 161 – PLATE TECTONICS

### STEREONETS

Some of the homework assignments involve various operations on vectors and planes, using a stereographic projection. Attached is a description of how this projection is constructed and how vectors and planes on the sphere are projected onto it. Finally, a description of how to perform some key stereonet operations is provided for your reference.

You may be familiar with two kinds of stereonets: equal-angle (stereographic projection) and equal-area (azimuthal equal-area projection). In this class, we will use the stereographic projection, but most of these operations can be performed in a similar fashion on an azimuthal equal-area projection.

In all of these operations, you will put an upside-down thumb tack through the center point of the stereonet and lay a piece of tracing paper over the stereonet. This way you will be able to mark points on the tracing paper and spin them around the center of the projection until you have a convenient alignment of great circles for the particular operation you are trying to perform.

Some definitions:

Great circle: Intersection between the earth's surface and a plane passing through the center of the earth. Lines of constant longitude on the earth's surface are great circles. The shortest distance between two points on a sphere is called a "great-circle path".

Small circle: Intersection between the earth's surface and a plane that does not pass through the center of the earth. Lines of constant latitude are small circles, except for the equator, which is a great circle.

#### LABELING

(1) UPPER HEMISPHERE PROJECTION: *Label stereonet so that you are looking at the outside of the earth's surface, seeing one hemisphere, centered on any longitude.*

Draw in the perimeter of the stereonet circle. Mark the top as the north pole ( $90^{\circ}\text{N}$ ) and the bottom as the south pole ( $90^{\circ}\text{S}$ ). Small circles of latitude are the horizontal or nearly horizontal curves, with the equator the horizontal line in the middle. Great circles of longitude are the fanning set of curves extending from the north pole to the south pole. To center the projection at ( $0^{\circ}\text{N}$ ,  $150^{\circ}\text{E}$ ), the vertical straight line will be  $150^{\circ}\text{E}$ . Lines of constant longitude will be counted west (left) from this center line; since it's  $90^{\circ}$  to the left side of the map projection, this line will be the  $60^{\circ}\text{E}$  great circle. The right side of the map projection will be  $150^{\circ}\text{E}+90^{\circ}\text{E} = 240^{\circ}\text{E}$  longitude or  $-120^{\circ}\text{W}$  longitude. This corresponds to looking at the hemisphere containing Asia and Australia.

(2) LOWER HEMISPHERE PROJECTION: *Label stereonet so that you are looking down at a point on the earth's surface, seeing the planes and vectors that pass through that point.*

Draw the four map directions – N, E, S, W – in clockwise fashion starting at the top. Or, you can number the azimuths clockwise from  $0^\circ$  to  $360^\circ$ .

## OPERATIONS

(1) *Plot the position of any vector on the stereonet.*

Remember, vectors plot as points on the stereographic projection.

Upper hemisphere projection: Plot a dot at the latitude and longitude of the point where the vector intersects the earth's surface.

Lower hemisphere projection: Tick off on the perimeter of the stereonet the azimuth corresponding to the trend of the vector. Rotate the tracing paper until this tick mark lines up with N or E. Count in from the perimeter of the stereonet the number of degrees equal to the plunge of the vector. Mark this position with a dot.

(2) *Plot the position of any plane on the stereonet.*

Remember, planes plot as lines. Planes passing through the center of the projection plot as great circles. Coordinates of planes will be specified by strike and dip for lower hemisphere projections. For upper hemisphere projections, great circles are specified either by their pole or by two lines that lie along them.

(3) *Given two vectors in space, find the plane containing them both.*

Plot the positions of the vectors as points on the stereonet. Rotate the stereonet until both plotted points lie on a single great circle trace. This great circle is the plane containing both vectors.

(4) *Given two vectors in space, find the angle between them.*

Plot the positions of the lines as points and find their common great circle as in (3) above. Count off along this great circle the number of degrees between the points. This is the angle between the vectors.

(4a) *Given two locations on the earth, find the distance between them.*

Proceed as in (4). Since one degree along a great circle on the earth equals 111.4 km, geocentric angle can be converted to great-circle distance.

(5) *Given a plane in space, find the vector that is perpendicular to it ("pole").*

Draw the trace of this plane as a great circle. Rotate the tracing paper until this great circle lines up with one of the great circles on the stereonet. Hold the tracing paper fixed and find the point where this great circle intersects the equator. Count  $90^\circ$  along the equator from this point towards (and across) the center of the stereonet. The point you find is  $90^\circ$  away from all points on the great circle and hence represents the vector perpendicular to the original plane. Note that in 3-space there are two antiparallel vector directions that could be perpendicular, but usually only one of these will fall within your stereonet (since the stereonet represents a hemisphere, not a sphere).

(6) *Given a plane  $P_1$  in space, find the plane  $P_2$  perpendicular to it that passes through a given vector  $L$ .*

Find the normal  $N$  to plane  $P_1$  as in (5). Plane  $P_2$  must contain  $N$  and also must contain the point representing the given vector  $L$ . Use (3) to find the plane  $P_2$ .

(7) *Given a vector  $L$  in space, find the plane  $P$  perpendicular to it.*

Plot the point representing  $L$  and rotate the tracing paper so that  $L$  lies on the equator. Count  $90^\circ$  away from  $L$  (passing through the center of the stereonet). The N-S great circle intersecting the equator at this position represents the plane  $P$ .

(8) *Given two planes in space, find the angle between them.*

Plot the poles to the planes as in (5). Find the angle between the poles as in (4). This will be equal to the angle between the perpendiculars to the planes. Note that each plane has two possible perpendiculars, so you can end up with either an acute angle or an obtuse angle as the angle between the planes, unless the planes are perpendicular to each other. Simpler, less accurate method (stereographic projection only): Draw both planes on the stereonet and draw tangents to them at the point where they intersect. Measure this angle with a protractor.

(9) *Given a great circle  $C_1$  on the earth, find its local azimuth at a specified point  $A$ .*

Draw  $C_1$  and plot  $A$ . Find the great circle  $C_2$  that passes through  $A$  and the north pole, and draw it. (This  $C_2$  obviously will have a local azimuth of  $0^\circ$  at point  $A$ ). Plot the poles to  $C_1$  and  $C_2$ . Find the angle between  $C_1$  and  $C_2$  as in (8). This angle will give you the local azimuth of  $C_1$  at  $A$ , but check to make sure the sign convention is correct.

Simpler, less accurate method (stereographic projection only): Draw  $C_1$  and plot  $A$ . Draw the great circle  $C_2$  passing through  $A$  and the north pole. Draw tangents to  $C_1$  and  $C_2$  at  $A$ . Use a protractor to measure the angle between them. This angle will be the local azimuth, but check to make sure that the sign convention is correct.

(10) *Given a location  $X$  on the earth, construct the great circle  $C_1$  passing through  $X$  at a given local azimuth  $\alpha$ .*

This is basically the reverse of (9). Draw the great circle  $C_2$  passing through  $X$  and the north pole. Find the normal to this great circle ( $N_2$ ). Draw the great circle  $C_3$  that is perpendicular to  $X$ , following (7). Note that  $N_2$  lies in  $C_3$ . Note that  $C_3$  contains all of the normals to planes that pass through  $X$  and hence must contain  $N_1$ , the normal to  $C_1$ . Find two points on  $C_3$  that are  $\alpha$  degrees away from  $N_2$ . One of these will be  $N_1$ , the normal to  $C_1$ . Draw the great circles perpendicular to these two points and look at them to determine which one has the correct sign of the local azimuth. Simpler, less accurate method (stereographic projections only): Draw the great circle  $C_2$  passing through  $X$  and the north pole. Draw a tangent to this circle at  $X$ , measure  $\alpha$  degrees from it with a protractor, and draw a straight line. Rotate the stereonet until this straight line lies along a meridian, and draw the corresponding great circle.

(11) *Given a vector in space (or point  $L$  on the earth), rotate it a specified amount about a specified pole of rotation  $P$  on the perimeter of the projection.*

Rotate the tracing paper so that  $P$  coincides with the north pole. Move  $L$  the specified number of degrees (check sign convention) by translating it along the correct small circle. Rotate the tracing paper back to its original position and read off the coordinates of the rotated point.

(12) *Given a point  $X$  on the earth, construct the small circle around it at a given distance  $\alpha$ . Count off  $\pm \alpha$  degrees along the great circle containing  $X$  and draw points there. Rotate the stereonet so that  $X$  falls on a different great circle. Count off  $\pm \alpha$  degrees from  $X$  along this great circle and draw points. If you are using a stereographic projection, the trace of the small circle will look like a circle on the projection, with these points on the perimeter. If you are using the equal-area projection, the trace of a small circle will not look like a circle on this projection, so you will have to repeat this procedure until you have enough points to interpolate the great circle.*

(13) *Rotate a point  $X$  on the earth by  $y$  degrees about a pole  $P$ , where pole  $P$  is not on the perimeter of the stereonet.*

You can do this either of two ways: following the directions in Cox and Hart's box 7.1 (pp. 222-224), or by using a drawing compass on an equal-angle stereonet. For the drawing compass technique, use (12) and construct the small circle about pole  $P$  that contains point  $X$ . Then, start at  $X$  and measure  $Y$  degrees along the small circle in the correct direction, and you will reach the rotated position of the point.

(14) *Find the vector  $X$  that represents the projection of an arbitrarily oriented vector  $B$  onto a plane  $P$ .*

When you project  $B$  onto the plane  $P$ , you decompose it into two perpendicular vectors, one of which is in plane  $P$  (vector  $X$ ) and the other of which is normal to plane  $P$  (vector  $N$ ). Because  $N$ ,  $B$ , and  $X$  all are coplanar, they all must lie in the same great circle. So, this is how you solve the problem:

Draw the great circle corresponding to plane  $P$ . Draw the normal  $N$  to the plane  $P$ . Draw the vector  $B$  on your projection. Construct the great circle containing  $N$  and  $B$ . The intersection of this great circle with plane  $P$  is the vector  $X$ .