Ge/Ay 132

Midterm/Problem set # 2

You are allowed to use the class notes and books to solve these problems. Limited collaboration is permitted. Don't hesitate to contact us if there are ambiguities in the questions. The questions are each worth 25 points. Due Friday, February 6th. Good luck!

1. The following table lists the energy differences, $\Delta E = E(J = 3/2) - E(J = 1/2)$ in cm^{-1} for the fine structure levels of the ground terms 3p $^2\text{P}^o_J$ of several ions of the Aℓ I isoelectronic sequence.

Fine Structure Splittings

Ion	Z	ΔE (cm^{-1})
Ti X	22	7542
V XI	23	9696
Cr XII	24	12261
Mn XIII	25	15320

a. Find an expression of the form $\Delta E = AZ^B$ that represents these data fairly well.
b. During the solar eclipse in 1869, an emission line was observed in the solar corona at a wavelength of 5303 Å. Armed with the empirical relation found under a), suggest a possible identification for this emission line.
c. Estimate very roughly the temperature at which this ion would be the dominant (i.e., most abundant) form of its element in thermodynamic equilibrium. Assume that the electron density in the solar corona is $N(e) \approx 1.2 \times 10^{10}$ cm^{-3}.

2. Atomic hydrogen has a ground–state photoionization cross section that is given approximately by $\sigma_H(\nu) = 6.3 \times 10^{-18}(\nu/\nu_o)^{-3}$ cm^2 for $\nu \geq \nu_o$, and σ_H=0 for $\nu < \nu_o$, where ν_o=109678.8 c Hz is the threshold frequency and c is the speed of light in cm s^{-1}. Assume that a dilute, uniform gas of density n=n(H) + $n(\text{H}^+)$ at temperature $T = 8 \times 10^3$ K with $n(\text{H}^+)$=$n(e)$ is exposed to isotropic radiation of intensity $J_\nu = 4\pi I_\nu = 10^{-20}(\nu/\nu_o)^{-1}$ ergs s^{-1} cm^{-2} Hz^{-1}. You may assume that all neutral atoms are in the ground state.

a. Calculate the rate of photoionization of H in s^{-1}.
b. Assume that photoionization of H is balanced by radiative recombination of H^+ and e with a thermal rate coefficient α_H= $1.9 \times 10^{-10}T^{-0.7}$ cm^3 s^{-1}. Calculate the fractional ionization x=$n(\text{H}^+)/n$ for n=0.005, 1.0 and 200.0 cm^{-3}.
c. Compute the corresponding ionization temperature in thermodynamic equilibrium that would give the same degree of ionization as in b).

3. The accompanying table is a curve of growth for the He I line at 3888.65 Å. The absorption oscillator strength is 0.06446. It arises from an excited long–lived state and it has been observed in absorption in the spectra of stars in the Orion nebula and in the Carina nebula.

a. What levels of He I are most likely involved? Why?
b. Write down the formula for the equivalent width of this line in the optically thin limit, and test its accuracy for (lower level) column densities of 3×10^{10}, 10^{12} and 3×10^{13} cm^{-2}.
c. Draw a graph of the curve of growth in the form log W_λ vs. log N.
d. The measured equivalent widths in the Carina Nebula are W=0.198, 0.098 and 0.058 Å. What are the corresponding column densities of He I in the lower level of the transition? If W is uncertain by 15%, what are the uncertainties in N (approximately)?
e. As d), but for W=0.8 Å.

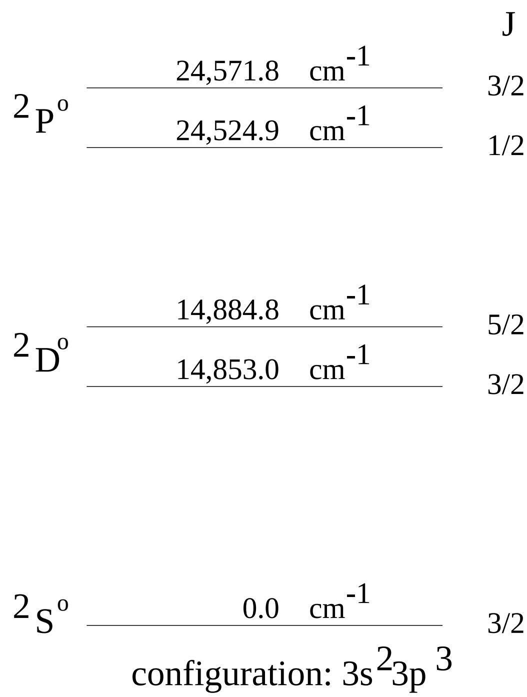
Curve of growth for He I 3888.65 Å for T_D=12500 K
Log N is in cm^{-2} and W_λ is in Å.

log N	W_λ	log N	W_λ
11.0	0.00086	16.5	0.7293
11.5	0.00272	17.0	0.7888
12.0	0.00852	17.5	0.8569
12.5	0.02621	18.0	0.9563
13.0	0.07611	18.5	1.145
13.5	0.1886	19.0	1.547
14.0	0.3387	19.5	2.375
14.5	0.4495	20.0	3.967
15.0	0.5335	20.5	6.891
15.5	0.6052	21.0	12.154
16.0	0.6694	21.5	21.553

4. Consider the sulfur ion S^+ with the configuration 3s^2 3p^3, and the corresponding terms ^{4}S, ^{2}D and ^{2}P. The energy levels are illustrated in the accompanying figure and the table below summarizes the relevant atomic data.

a. What is the critical density for each of the levels?
b. Set up the rate equations for the level populations for the 3–level system consisting of the ^{4}S, ^{2}D and ^{2}P terms (i.e., treat the ^2D_J and ^2P_J fine–structure levels as one level). Which terms in the equations are likely to be the dominant contribution to the populations of each of the levels?
c. Solve for the relative populations of the 3 levels assuming $T = 8 \times 10^3$ K and $n(e)$=100 cm^{-3}, taking only the most important terms into account.
d. What is the ratio of the $^2D_{5/2}$–$^4S_{3/2}$ and $^2D_{3/2}$–$^4S_{3/2}$ line intensities at low densities? And at high densities? In what regime is this line ratio most sensitive to density?

Transition	Upper J	Lower J	Ω	A (s^{-1})
$^2D^o$-$^4S^o$	5/2	3/2	4.19	2.60×10^{-4}
	3/2	3/2	2.79	8.82×10^{-4}
$^2P^o$-$^4S^o$	3/2	3/2	1.52	0.225
	1/2	3/2	0.759	0.0906
$^2D^o$-$^2D^o$	5/2	3/2	7.59	3.35×10^{-7}
$^2P^o$-$^2P^o$	3/2	1/2	2.38	1.03×10^{-6}
$^2P^o$-$^2D^o$	3/2	5/2	4.79	0.179
	3/2	3/2	3.38	0.133
	1/2	5/2	2.56	0.0779
	1/2	3/2	1.52	0.163



SII Energy Levels