Ge/Ay 132

Problem set # 1

You are allowed to use the class notes to solve these problems, but no other books. The questions are worth 20, 20, 30 and 30 points, respectively. Due Friday, January 23rd. Please discuss any need for an extension with one of us (Geoff or Susanna) in advance. Limited collaboration is permitted. Don't hesitate to see us if you find ambiguities. Corrections or clarifications will be posted on the Ge/Ay 132 web site.

1. Consider the ion $^{12}C^{5+}$ (C VI).

a) What is the value of the Rydberg constant for this ion, using the most accurate values of the fundamental constants?

b) Compute the (vacuum) transition frequencies and (air) wavelengths of the first three lines of the Balmer and Pfund series, and the first two lines with n''=100, neglecting relativistic terms. Classically, how large is the n''=100 state? Air wavelengths and vacuum wavenumbers are related by the index of refraction, the form of which can be written:

$$\lambda_{\rm air}^{-1} = (1+n)\tilde{\nu}_{\rm vac}$$

where $\lambda_{\rm air}$ is in cm, $\tilde{\nu}_{\rm vac}$ is in cm^{-1}, and

$$(1+n) = 1.0 + 8.34213 \times 10^{-5} + \frac{2406030}{1.30 \times 10^{10} - \tilde{\nu}^2} + \frac{15997}{3.89 \times 10^9 - \tilde{\nu}^2}$$

in standard air (B. Edlen 1966, *Metrologia* **2**, 71).

c) Consider the energy levels for n=2. How large is the spin–orbit splitting in cm^{-1} for each of the levels? And for n=3?

d) Consider the Hα transition. Draw a schematic energy diagram of the fine–structure levels involved. What are the allowed electric dipole transitions between the various fine–structure levels? What is the appearance of the spectrum (i.e., what lines tend to group together)?

2. In the spectrum of Ca^+, the short–wavelength limit of the diffuse series is at 1417.3 Å. The lines of the first doublet in the principal series ($4^2P_{1/2} \rightarrow 4^2S_{1/2}$ and $4^2P_{3/2} \rightarrow 4^2S_{1/2}$) lie at 3968.464 and 3933.659 Å. (Note that spectroscopic notation puts the higher energy level first, then a backwards or forward arrow to indicate respectively absorption or, as in this case, emission.)

a) Draw a schematic energy level diagram of the lowest five terms.

b) Write the terms involved in the first doublet of the sharp series.

c) What is the spacing in cm^{-1} of this doublet?

d) Compute the first ionization potential of Ca^+ in cm^{-1} and eV.

3. Give the spectroscopic terms arising from the following configurations, using LS coupling. Include parity and J values. Give arguments for deriving the results.

a) 3p 4s
b) 4d 5d
c) $3d^2$ (Hint: only five terms of those found under b) remain)
d) $3p^4$ 4s
e) $2p^2$ 4p
f) $3p^3$ (Hint: only three terms of those found under e) remain. Give arguments which three are likely to remain, rather than writing out completely all ot the possible determinants!)

4. Consider the ionized oxygen atom with the ground configuration $1s^2 2s^2 2p^3$, and the excited configuration $1s^2 2s 2p^4$.

a) What is the order of the LS terms arising from the ground and excited configurations (see also problem 3)? What is the expected order of the fine–structure levels within each of the terms? Draw a schematic energy level diagram.
b) List all transitions that are electric dipole, magnetic dipole or electric quadrupole allowed between these states considering (i) strict LS coupling; (ii) LS coupling including spin–orbit interaction and configuration–interaction. We'll look at this ion later in the course, specifically in H II regions. Can O^+ (O II to astronomers) be easily detected in cold gas?