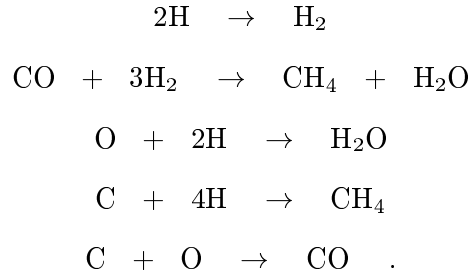


Ch/Ge128 – Problem Set #3
Due May 24th, 2005

(1) As was shown in class, the timescale for the depletion of gas-phase species onto grains in dark clouds is much shorter than the cloud lifetimes. (a) Using CO as an example, set up and solve a differential equation giving the CO abundance as a function of time assuming there are no mechanisms to desorb CO back into the gas phase. What is the timescale for depletion of CO, i.e., how long will it take for [CO] to fall by $1/e$? Express your answer in years. Take $n_{total} = 10^4 \text{ cm}^{-3}$, $[\text{grains}] \sim 10^{-12} n_{total}$, and $[\text{CO}] = 10^{-4} n_{total}$.

(b) Suppose the free fall time at an initial density of 10^3 cm^{-3} is 10^6 years. Plot the free fall time and depletion time for sticking coefficients of 0.1 and 1.0 for densities between 10^3 and 10^8 cm^{-3} .

(2) For the “simple” system involving H ($\Delta G = 48.58 \text{ kcal/mole}$), H_2 ($\Delta G = 0.0 \text{ kcal/mole}$), C ($\Delta G = 160.44 \text{ kcal/mole}$), CO ($\Delta G = -32.78 \text{ kcal/mole}$), CH_4 ($\Delta G = -15.60 \text{ kcal/mole}$), O ($\Delta G = 55.39 \text{ kcal/mole}$), and H_2O ($\Delta G = -54.63 \text{ kcal/mole}$); assume only the following reactions occur:



For total pressures of 0.1 and 1.0 atm and with $f(\text{H}, \text{H}_2) = 1$, $f(\text{C}) \sim 10^{-4}$, and $f(\text{O}) \sim 10^{-4}$, calculate the partial pressures of the various constituents at 10, 100 and 1000 K. What does this tell you about molecular clouds and the solar nebula?

(3) The table below gives the total mass, the “rock” mass, and the “ice” mass for the planets in units of Earth masses (For example, the Jupiter Mass of 320 means that the total mass of Jupiter is 320 Earth masses). Using the cosmic abundances of the elements, what is the minimal mass for the solar nebula which is required by the observed planets assuming the disk is composed of material of solar composition? For rock put all of the oxygen you can into silicates and other oxides; for ices use the remaining oxygen in water, and all of the available carbon and nitrogen in CH_4 and NH_3 (You can then calculate the abundances by mass which is what is necessary. The numbers below are clearly crude so don’t try and figure things out to 10%!). Express the result in terms of solar mass to compare the of the sun to the disk.

If this “minimum mass” nebula is spread out in a disk 10 AU in radius and 1 AU in thickness, using the ideal gas law, what is the nebular gas pressure at 500 and 1000 K?

Body	Total Mass	Rock Mass	Ice Mass
All Terrestrial	2	2	~0
Jupiter	320	5-15	5-20
Saturn	95	5-15	5-20
Uranus	15	2-4	~10
Neptune	17	2-4	~10

(4) The Jeans formula governing atmospheric escape due to thermal evaporation is:

$$\phi = n_i \langle v \rangle .$$

The flux of escaping particles where n_i is the number density of the species of interest and $\langle v \rangle$ is given by

$$\langle v \rangle = \frac{u}{2\pi^{1/2}}(1 + \lambda)e^{-\lambda} , \quad \lambda = \frac{GMm_i}{kTR} ,$$

where $u = (2kT/m_i)^{1/2}$, the most probable velocity of a Maxwell-Boltzmann distribution.

(a) What two forces compete to determine if a particle escapes, and how do they manifest themselves in the formula above?

(b) Given a solar composition for the primordial atmosphere of the earth, and assuming the atmosphere is well mixed, will H_2 and He escape? If so, how long will it take to deplete the atmosphere of 99% of each species? You can determine the initial abundance by assuming that the amount of N_2 has not changed. Assume $T = 200$ K.

(c) What is the flux from Jupiter of H_2 and He? (Again, at 200 K) How does this compare to the mass of each in the atmosphere, assuming solar composition, and what are their residence times (that is, the reservoir mass divided by the loss) if escape is constant and the dominant loss process?