How Many Plates?

By Don L. Anderson

California Institute of Technology

Seismo Lab

M/C 252-21

Pasadena, CA 91125

Tel: 626.395.6901

Fax: 626.564.0715

E-mail: dla@gps.caltech.edu

Word Count: 4,124

Abstract

The questions I address are: how many plates are there, and how many should there be? Is there a pattern in the plate mosaic? Related issues have to do with the optimal sizes and shapes of plates and spacings of ridges, trenches and transform faults. Similar questions arise in studies of foams, bubble rafts, buckyballs, mudcracks, columnar jointing, in the tessellation of spheres and the planforms of convection. In sphere covering problems, and in dynamic problems, pentagons replace the familiar hexagons. The "ground" state of plate tectonics on a homogeneous planet may involve about twelve plates with five nearest – and five next-nearest neighbors. The plate mosaic may be a self-organized network of plates and force chains which are readily reorganized by stress changes. This paper starts with the premise that the mosaic may have simple and surficial explanations rather than convective or plutonic causes. The study of the tessellation of the Earth can be called Platonics to distinguish it from the idea that the lithosphere necessarily mirrors the planform of mantle convection.

Keywords: tectonics, plates, geodynamics, self-organization

INTRODUCTION

Much of nature seems to organize herself with little attention to the details of physics. Large interacting systems tend to self-organize. Thus, mudcracks, honeycombs, frozen ground, convection, basalt columns, foams, fracture patterns in ceramic glazes and other natural features exhibit similar hexagonal patterns. In many cases there is a *minimum* or *economy* principle at work; minimum energy, area, stress, perimeter, work and so on. Dynamic systems, including plate tectonics, may involve dynamic minimization principles such as least dissipation. Natural processes spontaneously seek a minimum of some kind. These minima may be local; systems may jam before achieving a global minimum. Large complex interacting systems can settle down into apparently simple patterns and behavior.

In the Earth sciences polyhedra forms have been used to support contraction, expansion, and drift theories (Elie de Beaumont, 1829, Carey 1976, Spilhaus 1973). D'Arcy Thompson (1917) pointed out that identical forms can be generated by very different forces; it is impossible to deduce from observed patterns alone which forces are acting. The corollary is that pattern formation may be understood, or predicted, at some level without a complete understanding of the physical details.

The search for global patterns in surface topography and geology goes back to Elie de Beaumont (1829). Dodecahedra, icosahedra and icosidodecahedra have all been proposed for the underlying structure of global features (e.g. Spilhaus, 1973, Sears, 2001). This mathematization of geology is viewed with amusement by historians (Oreskes, 1999) and emphasis has shifted to modeling of mantle convection. These fluid dynamic simulations have not been successful in producing plate tectonics (e.g. Bercovici et al, 2000) and the most sophisticated treatments put in the observed plate configurations and motions as boundary conditions. The basic question then remains.

We may ask the question in the following way:

In the ideal (Platonic) plate-tectonic world what is the optimal shape and size of a tectonic plate and how many plates are there?

The shapes of buckminsterfullerenes, radiolaria and viruses are spherical tessellation issues and are not yet fully understood. They are related to the tiling of a sphere, and minimum perimeter problems. The optimal arrangement of tiles on a sphere has a correspondence with many distinct physical problems (carbon clusters, clathrates, boron hydrides, quasicrystals, distribution of atoms about a central atom and bubbles in foam). It is often found that straight lines (great circles) and equant cells (squares, pentagons or hexagons rather than rectangles) of identical size serve to minimize such quantities as perimeter, surface area, energy and so on. There is an energy cost for creating boundaries and larger entities often grow at the expense of smaller ones. In phenomena controlled by surface tension, surface energy, stress, elasticity and convection one often finds tripartite boundaries (called triple junctions or valence-3 vertices) and hexagonal patterns. The classic problem of convection goes back to experiments by Bénard in 1900. The hexagonal pattern he observed was attributed to thermal convection for many decades but is now known to be due to variable surface tension at the top of the fluid. Plan views of foams also exhibit this pattern. In all of these cases space must be filled and principles of economy are at work.

IN THE REAL WORLD

Table 1 gives the parameters of the plates. Plate tectonics on Earth, at present, consists of about a dozen large semi-rigid plates, of irregular shapes and sizes that move over the surface, separated by boundaries which meet at triple junctions. There are also many broad zones of deformation. The seven major plates account for 94% of the surface area of the Earth. Gordon (2000) recognizes 20 plates and an equal number of broad zones of deformation. To the usual plates he adds Borneo, Capricorn, Caroline, Indo-China, Nubia, North China, Okhotsk, Somalia, Yangtze and Tarim. Asia and North America are collages of accreted terranes and the large Pacific plate grew by annexing neighboring plates (Hardeback and Anderson, 1996).

Three of the larger plates – AFR, IND and EUR – have large fractions of their areas occupied by diffuse deformation zones and do not qualify in their entirety as *rigid* plates. In fact, at least 15% of the Earth's surface violates the rules of rigid plates and localized boundaries. It is interesting that packing of similar sized polygons or circles on a sphere leaves about 15% void space (except when n is 6 or 12) (see Figure 1). The minor plates, in aggregate, are smaller than the smallest major plate. In a close-packed or random assemblage of discs the number of contacting neighbors decreases as the size disparity of the discs increases. At some point the system looses rigidity, typically at 15% void space. Six-coordinated structures tend to be rigid. Five-fold coordination is rare in rigid crystals but is common in fluids and glasses, on the surface of spheres, and at foam dislocations.

The Indo-Australian plate is divided into an Indian, Australian and Capricorn plate and diffuse zones of compression (Gordon, 2000). There are few earthquakes or volcanoes to mark these boundaries. The Pacific plate has several bands of earthquakes and volcanoes that could be cited as possible diffuse or incipient plate boundaries. One of these zones extends from Samoa and Polynesia to the East Pacific Rise and includes most of the "intraplate" earthquakes and active volcanoes in the Pacific. There is little evidence for relative motion between the north and south Pacific plates but current motions between EUR, ANT, and AFR are also very slow

(Gordon, 1995). Therefore, there are somewhere between 8 and 20 plates with 12 being a frequently quoted number. Plate reconstructions in the past also recognize about 12 persistent plates.

OTHER STATISTICS

The coordination numbers of the present plates are as follows:

i. nearest neighbors (NN) 4.8±1.2

ii. next—nearest neighbors (NNN) 5.3±0.6

These numbers were derived by counting all NN and NNN, with and without various of the minor plates, and then averaging. This process is robust to alternate choices of "official" plates. These statistics are similar to those found in coarsening 2D soap froths (Weaire and Hutzler, 1999). NN is 6 for a static equilibrium foam.

Ridges and trenches account about equally for 80% of the plate boundaries, the rest being transform faults. Most plates are not attached to a slab or bounded by a transform. Therefore, in the real world, plates are not equal, in either size or configuration.

A question arises; is the present situation typical, or more representative of a transitional state? Does active plate tectonics require a number of small "buffer plates" as well as the larger plates? If the coordination numbers of the plates increased, would the system jam or lock up?
In a world of rigid non-subducting plates, the surface would lock-up at about 15% "porosity". Most plates are not attached to slabs and their freedom to move is constrained by the surrounding plates.

BUBBLES AND MINIMAL SURFACES

Foams are collections of surfaces which minimize area under volume constraints. The four basic confined bubble structures have 12 pentagonal faces and are composed of a basic pentagonal dodecahedron with two, three or four extra hexagonal faces (Sullivan, 2000). These structures have only pentagonal and hexagonal faces, with no adjacent hexagons. Clathrates and zeolites also adopt these structures. Carbon cages involve the basic pentagonal dodecahedron unit supplemented by hexagonal faces with an isolated pentagon rule for the bigger molecules. These structures are constrained minimal energy surfaces and, may serve as models for the tessellation of the Earth. All vertices of these structures are triple-junctions, meeting at 120°; all faces are pentagons or hexagons. Sheared bubbles in foams can adopt complex shapes, such as boomerangs, but they are still minimal surfaces (Weaire and Hutzler, 1999). In close–packed arrays of bubbles the dominant hexagonal coordination is interrupted by linear defects characterized by five–coordination. The midpoints between objects packed on sphere often define a pentagonal network. Even on a plane, random dense packings involve pentagonal networks and this is more likely on a sphere, even for optimal dense packings (Tarnai and Gaspar, 1991, 2001).

From analogous geometric problems (foams, Marangoni convection, buckyballs, clathrate structures, tiling of spheres) I suspect that the ideal plate will be bounded by 5 or 6 edges and that plate boundaries will approximate great circles which terminate at triple junctions dominated by 120° angles (Figure 2).

JAMMING

Bubble rafts, or 2D foams, are classic minimum energy systems and show many similarities to plate tectonics. They are examples of *soft matter*. They readily deform and *recrystallize* (coarsen). Foams are equilibrium structures held together by surface *tension*. A variety of systems, including granular media and colloidal suspensions, exhibit non-equilibrium transitions from a fluid-like to a solid-like state characterized by jamming of the constituent particles (Trappe et al, 2001). The jammed solid can be refluidized by thermalization, temperature, vibration, or by an applied stress. These are termed *fragile* media (Cates et al, 1998).

Granular material and colloids tend to self-organize so as to be compatible with the load on them. They are held together by *compression*. They are rigid or elastic along compressional *stress chains* but they collapse and reorganize in response to other stresses, until they jam again in a pattern compatible with the new stresses. The system is weak to incompatible loads. Changes in *porosity*, temperature or stress are equivalent and can trigger reorganization and apparent changes in rigidity. The jamming of these materials prevents then from exploring phase space so their ability to self-organize is restricted, but is dramatic when it occurs.

In the plate tectonic context it is compression that keeps plates together. When the stress changes, and before new compatible stress circuits are established, plates may experience extension and collapse. New compatible plate boundaries must form. Widespread volcanism is to be expected in these un-jamming and reorganization events. These events are accompanied by changes in stress and in the locations and nature of plate boundaries and plates, rather than by abrupt changes in plate motions. Volcanic chains, which may be thought of as chains of tensile stress, will reorient, even if plate motions do not. Jamming theory may be relevant to plate sizes, shapes and interactions. The present plate mosaic is presumably consistent with the stress field that formed it but a different mosaic forms if the stresses changed.

The Platonic philosophy distinguishes between the real world and the ideal world; the world that is, and the world that it will become. The principle that controls the shapes and numbers of plates I call Platonics. The principle is not necessarily thermal convection in the interior of the planet (plutonics) but may be one of self-organization.

IS THERE A PATTERN?

The two largest plates (PAC, AFR) are antipodal and are surrounded by a band of intermediate sized plates and geoid lows (Anderson, 1989). An equal area projection centered on Africa emphasizes the pentagonal shape of the African plate and the symmetry of the surrounding polygons (Spilhaus, 1973). Figure 3 shows an unwrapped globe where each plate is an equal area projection.

The most remarkable property of plates is that most of them have five nearest ("kissing") neighbors *and* five next-nearest neighbors. This is the coordination of a pentagonal dodecahedron. This relation holds in spite of the fact that the plates are far from regular pentagons in shape and size. Bubbles in foams and bubble rafts also show little dispersion from the mean values of 5-6 nearest neighbors in a plane, in spite of variations in size and shape. Foams also have the familiar 120° triple junctions which evolve and annihilate (Weaire and Hutzler, 1999). The optimal packings of circular caps on a sphere involves 5 or less kissing neighbors (Tarnai and Gaspar, 1991; Fejes-Tóth, 1964; Fowler and Tarnai, 1996). Packings of pentagons on a sphere involve five NN (Tarnai and Gaspar, 2001).

IN THE IDEAL WORLD

> *The ideal world may have n identical faces (plates, tiles) bounded by great circle arcs which meet three at a time at 120°.*

This simple conjecture dramatically limits the number of possibilities for tessellation of a sphere and possibly, for the ground state of plate tectonics. In soap bubbles and plate tectonics, junctions of four or more faces are unstable and are excluded. There are ten such possible networks of great circles on a sphere (Taylor and Gladbach, 1976). Some of these are shown in

Figure 2. If plate boundaries approximate great circles meeting three at a time at 120° then there can be a maximum of twelve plates.

The study of convective planforms and pattern selection is a very rich and fundamental field (Bercovici, 2000). Even in complex convection geometries regular polyhedral patterns are common. Pattern selection in the plate tectonics system, however, may have little to do with an imposed pattern from mantle convection although it is commonly assumed to do so. Of the ten ways of drawing arcs of great circles on a sphere so that all intersections are at 120°, eight are equilibrium configurations for soap bubbles. Five of the shapes are regular in that they have identical faces; circular caps, two sided slices, triangles, squares or pentagons.

The optimal arrangement of spheres is a classical topological problem. The best packing in 2D is the familiar hexagonal lattice. The fraction of the plane occupied by circles is 0.9069… Packing circles on a sphere depends on the number of circles (spherical caps). The area covered ranges from 0.73 to 0.89 for $n \leq 12$ and oscillates about 0.82 at least up to $n = 80$ (Figure 1). Packing of more than 6 regular tiles on a sphere is inefficient except for 12 equal spherical pentagons which can tile a sphere with no gaps. The efficiency of packing, the sizes of the voids and their aggregate area depends little on the size distribution, within limits (Fejes-Tóth, 1964). The voids between regular tiles on a sphere, when close packed, are typically 10% of the radius of the discs. Typically, equal sized circular or polygonal non-overlapping caps can cover only about 70 to 85% of the surface of a sphere. About 15% void space occurs even for optimal packing of large numbers of circular or pentagonal caps. However, one can efficiently arrange 12 caps onto a sphere, with only 0-10% void space. This is much more efficient than for, say, 10, 13, 14 or 24 caps. Furthermore, the difference between the sizes of caps which pack most efficiently (least void space), and cover the whole surface most economically (least overlap), is relatively small for 12 caps (the difference is zero for regular spherical pentagons).

Since plates are held together by networks of compressional forces it is important that they pack efficiently. On the other hand, they must be mobile, and cannot be a permanently jammed system. Materials with this rigid-fluid dichotomy are called *fragile* (Cates et at, 1998). Close-packed networks of objects are *jammed* or *rigid*. However, even open networks can jam by the creation of load bearing *stress chains* (Cates et al, 1998) which freeze the assemblage so it cannot minimize the open space. These networks can be mobilized by changing the stress.

The least dense closest packings for similar size and shape caps on a sphere is about 70% coverage. The deformed zones along island arcs, and the strips of thin hot lithosphere along ridges add up to about 15% of the surface area of Earth which gives a total of about 30% when added to Gordon's (2000) broad deformed zones. In principle, this could still jam considering the tradeoffs between *porosity*, stress and temperature (Trappe et al, 2001). Jamming represents a self-organized network of stress chains.

DYNAMICS

How might the forces acting on the surface of a planet tend to subdivide it?

The forces may involve thermal contraction, "slab pull", "ridge push", tearing (changes in dips of bounding slabs), stretching (changes in strike of boundaries), bucking, flexure, convection, jamming, force chains and so on. In far-from-equilibrium systems the minimizing principle, if any, is not always evident. The first step in developing a theory is recognition of any patterns and identification of the rules. To date, most of the emphasis on developing a theory for plate tectonics has involved thermal convection, driven from within and below. If plates drive and organize themselves, and organize mantle convection, or if plates are rigid and can jam, then a different strategy is needed. The mechanical integrity of a plate probably results purely from the applied load (gravity).

The definition of a *plate* and a *plate boundary* is subjective. Things are constantly changing. It is improbable that there is a *steady-state* or *equilibrium* configuration of plates since only a few of the 16 kinds of triple junctions are stable. Nevertheless, there does seem to be a pattern in the plate mosaic and there are similarities with so-called minimal surfaces. From strictly static and geometric considerations the ideal (Platonic) world may consist of 12 tiles with 5 NN and 5 NNN coordinations. The real dynamic world appears to contain a hint of this ideal structure (Spilhaus, 1973). A regular pentagonal dodecahedron with rigid faces would be a jammed structure, (Plato argued that true knowledge was forever fixed in the ideal realm of Platonic forms.) Fewer, or more, plates, deformable plates and a bimodal distribution may be required to mobilize the surface mosaic. Vibrations (earthquakes) temporarily mobilize sections of plate boundaries.

DISCUSSION

On a homogeneous sphere one expects that surface tessellations due to physical processes will define a small number of identical domains. This will be true whatever the organizing mechanism unless symmetry breaking and bimodal domains are essential for the operation of plate tectonics. There are only a few ways to tessellate a sphere with regular spherical polygons that fill space and are bounded by geodesics and triple junctions. Whether the surface is subdivided into "faces" by deep mantle convection forces or by superficial (Platonic) forces we expect the surface of a homogeneous sphere to exhibit a semi-regular pattern, although this pattern will not be stable if the triple junctions are not.

In contrast to the beehive and other isoperimetric problems, and the bubble problems, we do not know what is minimized, if anything, in plate tectonics. It could be boundary or surface energy, toroidal energy or dissipation. Nevertheless, it may be fruitful to approach plate geometry initially as a tiling, packing, or isoperimetric exercise.

The problems of tessellations of spheres and global tectonic patterns are venerable ones. No current theory addresses the issues raised in this paper. The planform of a freely convecting spherical shell may have little to do with the sizes, shapes and number of plates. The plates in fact may self-organize and serve as the template that organizes mantle flow.

Table 1 Parameters of Plates

Plate		**Area**	**Growth rate**		
		10^6 km^2	km^2/100 y		
Pacific	PAC	108	-52	*	O
Africa	AFR	79†	+30	*	C
Eurasia	EUR	69†	-6		C
Indo-	INA	60†	-35	*	C
N. America	NAM	60†	+9		C
Antarctic	ANT	59	+55		C
S. America	SAM	41†	+13		C
Nazca	NAZ	15	-7	F	O
Arabia	ARA	4.9	-2	AF	C
Caribbean	CAR	3.8	0	**	O
Cocos	COC	2.9	-4	F	O
Philippine	PHI	~5		**	O
Somali	SOM			AF	C
Juan de Fuca	JdF			F	O
Gorda	GOR			F	O
Scotia	SCO			**	O
SE Asia	SEA			**	C
Indian	IND				C

F Farallon plate residues
AF AFR fragments
* possibly two or more plates
** buffer plates?
O mainly oceanic

† these plates are substantially reduced in area if the broad deformational zones are removed (Gordon, 2000)
C mainly continental

FIGURE CAPTIONS

1. When tiles of a given size and shape are packed on a sphere, without overlap, the packing density depends on the number *n*. The optimal packing density is achieved for *n*=12. This figure is for spherical caps but other shapes give similar results. The voids between the tiles have dimensions of order 10% of the tile dimensions so only small tiles or plates can be accommodated in the interstices. (After Clare and Kepert, 1991).
2. There are ten possible configurations of great-circle arcs on a sphere that meet three at a time with angles of 120°. Six of these are shown here. Only five have identical faces. These represent possible optimal shapes of plates and plate boundaries. These figures have 2,3,4,6,10 and 12 faces or plates (after Taylor and Gladbach, 1976).
3. A disjointed Lambert equal-area projection centered on each plate. Note the band of medium sized plates encircling the large African and Pacific plates. Note the similarity in shape of AFR and PAC, and of EUR and INA.

Figure 1

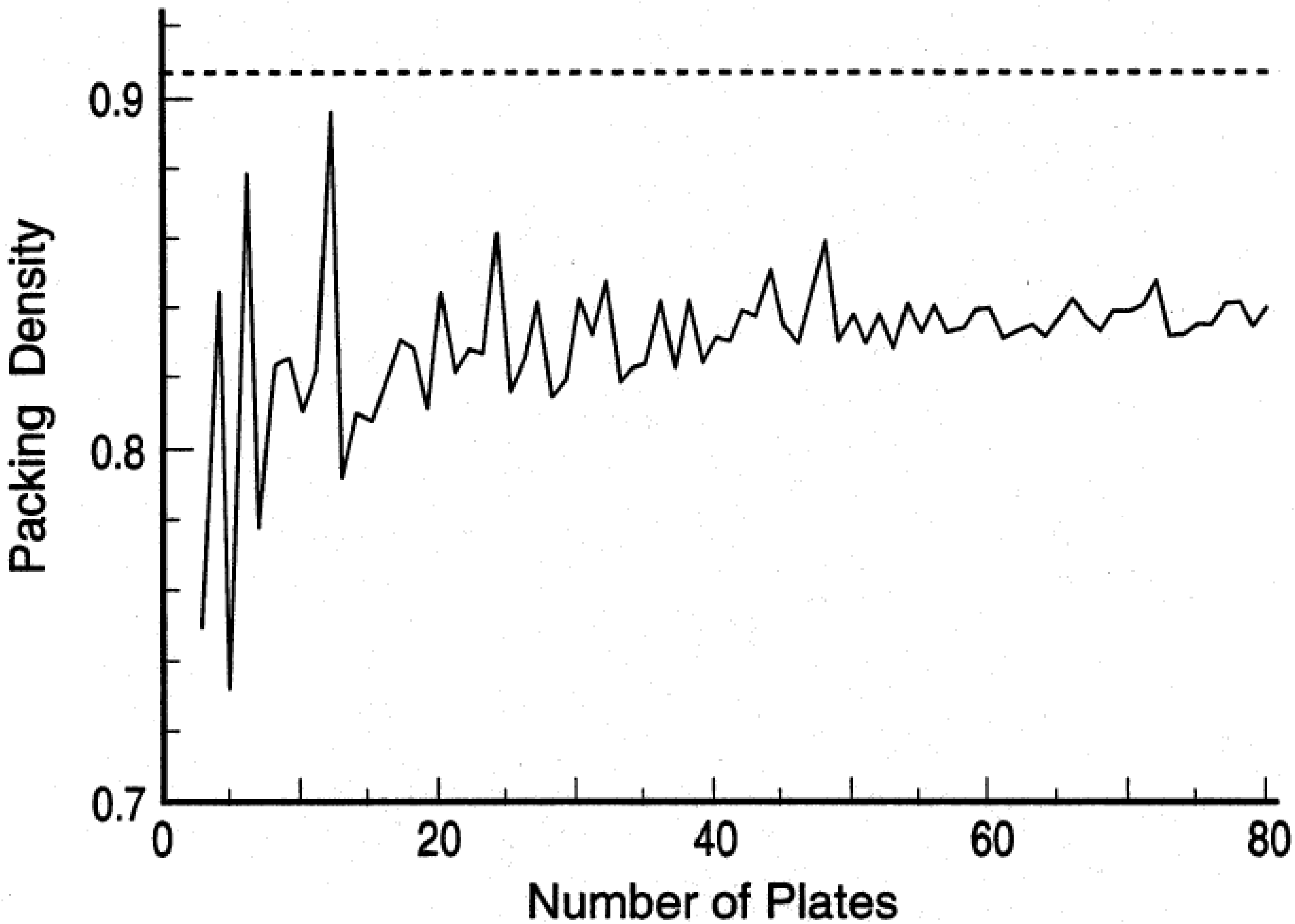


Figure 2

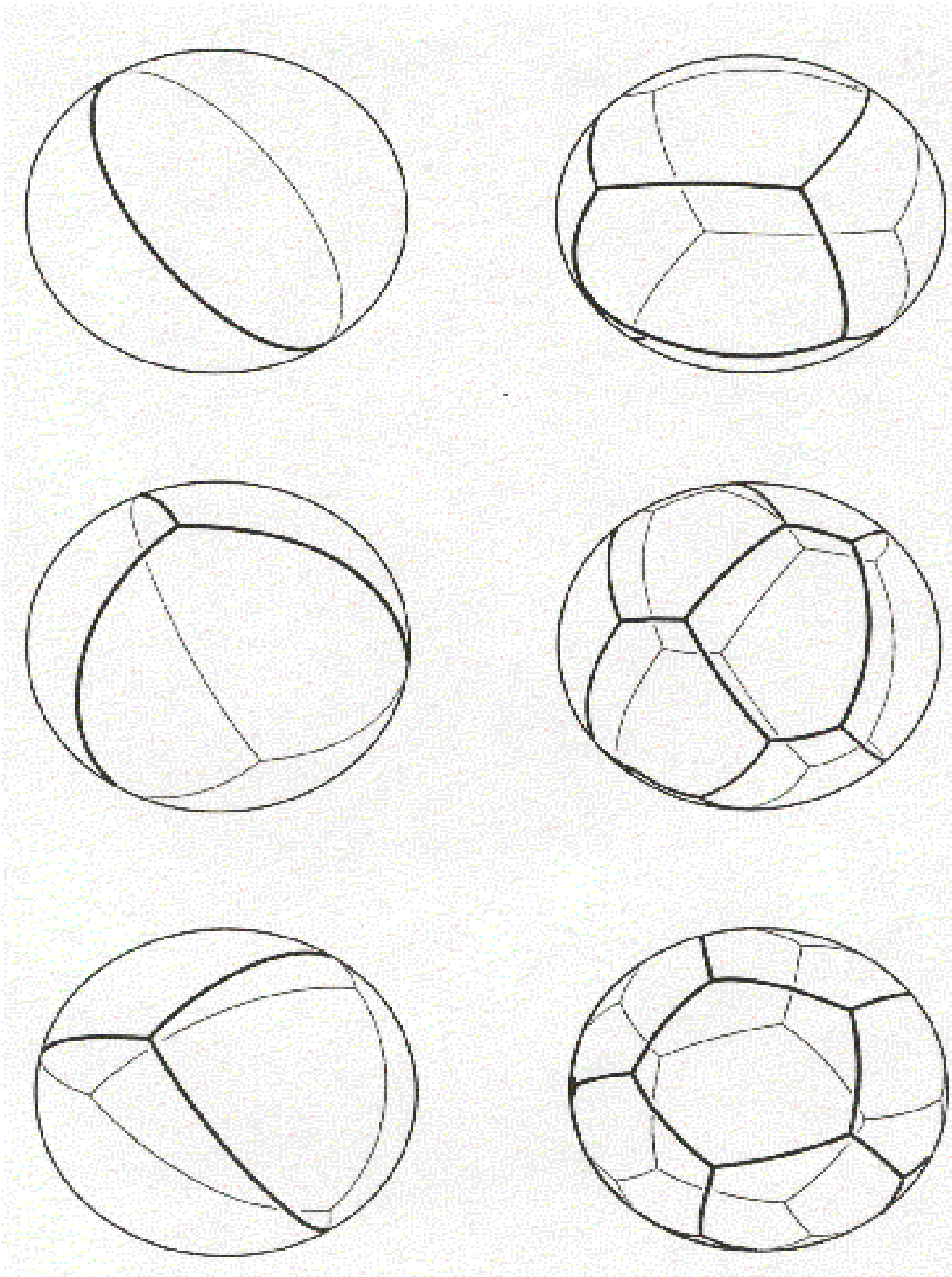
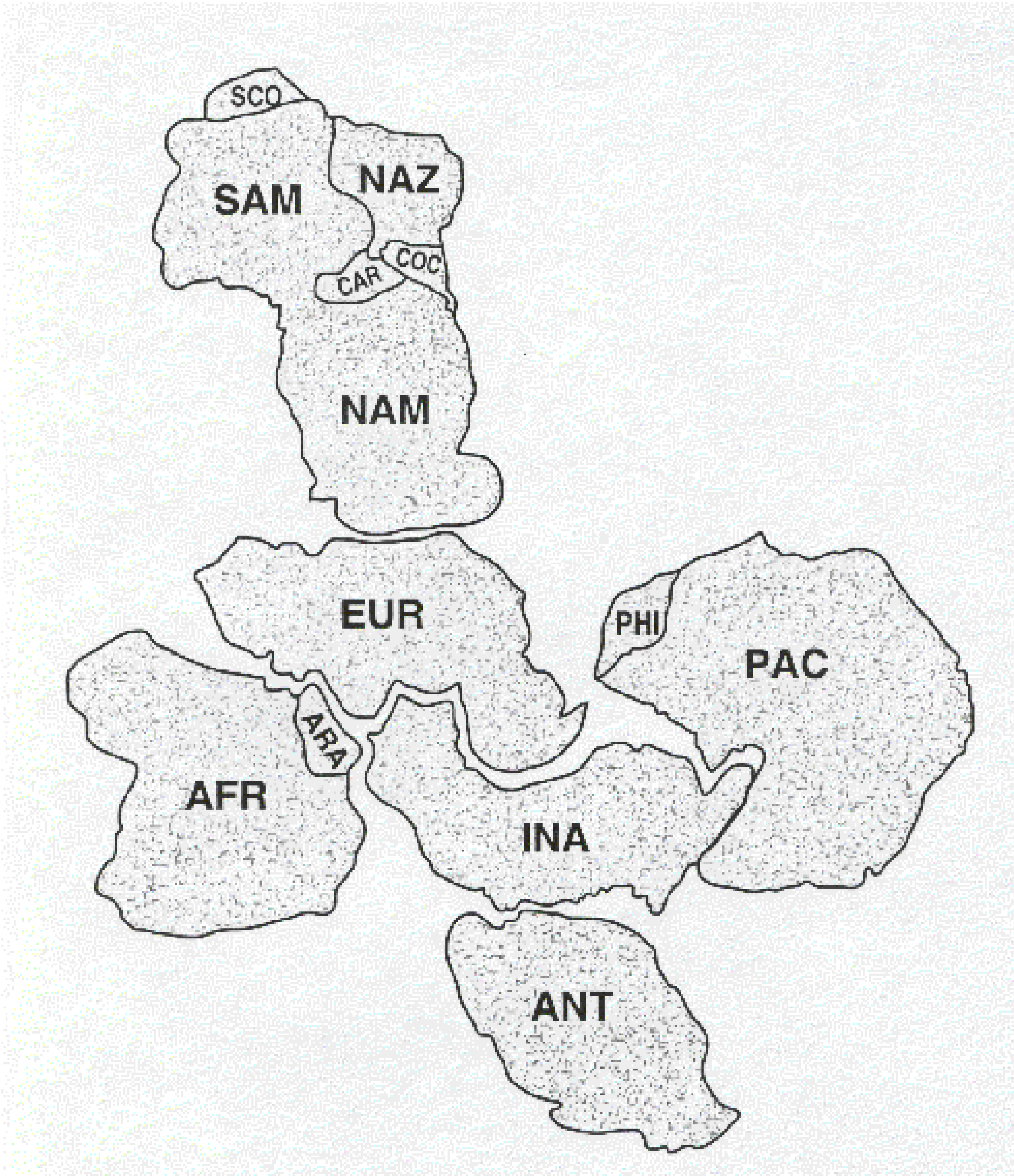


Figure 3



REFERENCES CITED

Anderson, D.L., 1989, Theory of the Earth, Blackwell Scientific Publications, Boston, p. 366.

Bercovici, D., Ricard, Y., and Richards, M., 2000, The relation between mantle dynamics and plate tectonics: a primer, *in* The History and Dynamics of Global Plate Motions, ed. Richards, M., Gordon, R., and van der Hilst, R., Geophysics Monograph 121, AGU, Washington D.C., p.398.

Carey, S. W., 1976, The Expanding Earth, New York: Elsevier, p. 488.

Cates, M. E., Wittmer, J. P., Bouchaud, J.-P. and Claudin, P., 1998, Jamming, Force Chains and Fragile matter, Phys. Rev. Lett. , v. 81, p. 1841-1844.

Clare, B. W., and Kepert, D. L., 1991, The optimal packing of circles on a sphere: Journal of Mathematical Chemistry, v. 6, no. 4., p. 325-349.

Elie de Beaumont, L., 1829, Notice Sur les Systémes des montagnes: French Academy of Sciences, Paris.

Fejes Tóth, L., 1964, Regular figures, by L. Fejes Tóth: A Pergamon Press book: Macmillan, New York, p. 327-332.

Fowler, P. W., and Tarnai, T., 1999, Transition from circle packing to covering on a sphere: the odd case of 13 circles: Proceedings of the Royal Society of London Series A-Mathematical Physical and Engineering Sciences, v. 455, no. 1992, p. 4131-4143.

Gordon, R. G., 2000, Diffuse Oceanic Plate Boundaries, *in* The History and Dynamics of Global Plate Motions, ed. Richards, M., Gordon, R., and van der Hilst, R., Geophysics Monograph 121, AGU, Washington D.C., p.398.

Gordon, R. G., 1995, Present plate motions and plate boundaries, in Global Earth Physics, AGU Reference Shelf I ed. Ahrens, T. J., AUG Washington, D. C., p. 66-87.

Hardebeck, J. and Anderson, D. L., 1996, Eustasy as a test of a Cretaceous Superplume Hypothesis, Earth and Planetary Science Letters, v. 137, p. 101-108.

Kottwitz, D. A., 1991, The densest packing of equal circles on a sphere: Acta Crystallographica Section A, v. 47, part 3, p. 158-165.

Oreskes, N., 1999, The rejection of continental drift: Oxford University Press, p. 420.

Sears, J. W., 2001, Icosahedral Fracture Tessellation of Early Mesoproterozoic Laurentia, Geology, v. 29, p. 327-330.

Spilhaus, Athelstan, 1973, Geo-Art: Plate Tectonics and Platonic Solids: American Geophysics Union, Trans. 56, p. 52-57.

Sullivan, J. M., 2000, New Tetrahedrally Close-Packed Structures, Proceeding Eurofoam 2000, 3, p. 111-119.

Tarnai, T., and Gaspar, Z, 2001, Packing of equal regular pentagons on a sphere: Proceedings of the Royal Society of London Series A-Mathematical Physical and Engineering Sciences, v. 457, no. 2009, p. 1043-1058.

Tarnai, T., and Gaspar, Z, 1991, Covering a sphere by equal circles, and the rigidity of its graph: Mathematical Proceedings of the Cambridge Philosophical Society, v. 110, part 1, p. 71-89.

Taylor, J. E. and Gladbach, M. C., 1976, Scientific American, v. 235, p. 82-93.

Thompson, D'Arcy, 1917, On Growth and Form, Cambridge, p. 346.

Trappe, V., Prasad, V., Cipelletti, L., Segre, P. N. and Weitz, D. A., 2001, Jamming phase diagram for Attractive particles, Nature, v. 411, pp. 772-775.

Weaire, D. and Hutzler, S., 1999, The Physics of Foam, Clarendon Press, Oxford, p. 246.