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2 **Auxiliary Material of “A window into the complexity of the dynamic rupture of**
3 **the 2011 Mw 9 Tohoku-Oki earthquake”**

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12 **1. Movies of back-projection results**

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14 The attached files, *jp_movie_EU_music.gif* and *jp_movie_US_music.gif*, are
15 movies showing results of our MUSIC back-projection source imaging in the 0.5-1
16 Hz band. The quantities plotted and conventions are similar to those in Figure 3 of the
17 main text.

18

19

20 **2. Parameters of the array processing**

21 *theory*

22 We applied a high resolution array processing technique called Multiple Signal

23 Classification (MUSIC) [*Goldstein and Archuleta*, 1991; *Schmidt*, 1986], which
 24 estimates the direction of arrival (DOA) of multiple waves propagating through an
 25 array by a subspace projection method. We first estimate the covariance matrix of the
 26 array data for each sliding time window and each frequency using a multitaper
 27 technique. **Error! Reference source not found.** The eigenvectors corresponding to its
 28 largest eigenvalues span the signal subspace. Its complement defines the noise
 29 subspace. For a given DOA, a steering vector \mathbf{a} collects the Fourier-domain phase
 30 shift terms for each station resulting from the differential travel times relative to the
 31 hypocentral travel times. The MUSIC pseudo-spectrum, as a function of DOA, is
 32 defined as the inverse of the projection of the steering vector onto the noise subspace
 33 (through a projection matrix \mathbf{U}_n):

$$34 \quad P_m = \frac{\mathbf{a}^H \mathbf{a}}{\mathbf{a}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}} \quad (1)$$

35 where the superscript H denotes the complex conjugate transpose. Ideally, the
 36 steering vector of a signal is orthogonal to the noise subspace. In practice, the DOAs
 37 of the multiple sources are identified by the largest peaks of the MUSIC
 38 pseudo-spectrum. We improve the estimate robustness by averaging over a frequency
 39 band the pseudo-spectra normalized by their respective maximum at each frequency.
 40 The absolute amplitude of the MUSIC pseudo-spectrum is not directly related to the
 41 strength of the source. Once the DOA of the sources are determined, we estimate their
 42 power by the projection of the principal eigenvalues (diagonal matrix $\mathbf{\Lambda}_s$) onto the
 43 signal subspace (projection matrix \mathbf{U}_s) excluding the noise noise power (square of the
 44 averaged noise eigenvalues σ) [*Mestre et al.*, 2007]

45
$$P_w = \frac{a^H a}{a^H U_s (\Lambda_s - \sigma^2 I)^{-1} U_s^H a} \quad (2)$$

46

47 *Data pre-processing and selection*

48 We band-pass filtered the vertical component of the waveforms between 0.5 and 1 Hz.

49 We then selected the stations with signal to noise ratio greater than 3. We aligned

50 these on the first 8 seconds of P arrival by multi-channel cross-correlation [*Vandecar*

51 *and Crosson, 1990*]. The seismograms are then normalized by the standard deviation

52 of the first 200 s of P wave recordings.

53

54 *Length of sliding windows*

55 To achieve a balance between temporal resolution and imaging stability we fixed the

56 length of the analysis windows to 5 times the longest period at which the data is

57 analysed. For our primary frequency band (0.5-1 Hz) we slide a 10 seconds long

58 analysis window by steps of 1 second.

59

60 *Number of principal eigenvalues in MUSIC*

61 In theory the number (M) of eigenvalues of the covariance matrix contained in the

62 signal subspace is equal to the number of signal sources. In practice the signal is not

63 perfectly coherent and its power is partially diverted into the subspace of smaller

64 eigenvalues. We set M equal to the number of eigenvalues greater than the number of

65 stations times the floating point precision scaled by the largest eigenvalue.

66

67 *Number of tapers in cross-spectrum estimation*

68 MUSIC achieves better resolution than beamforming in direction of arrival estimation
69 problems with sufficiently long and stationary signals [Krim and Viberg, 1996].

70 However, seismic signals are highly transient. This hampers the robust estimation of
71 the data covariance matrix. To circumvent this issue, we estimate the array covariance
72 with the multitaper cross-spectrum estimator [Thomson, 1982], which averages the

73 Fourier estimates tapered by a sequence of functions with optimal spectral

74 concentration. The Slepian tapers maximize the ratio of energy within and outside a

75 frequency band W for a window of duration T . We require the frequency smearing

76 bandwidth to be at most 20% of the highest frequency (1 Hz), that is, $W = 0.2$ Hz.

77 Following standard practice, we use the first $2TW - 1$ tapers. Hence, for the

78 frequency band 0.5-1 Hz and $T = 10$ s we use 3 tapers.

79

80 **3. Foreshocks and aftershocks**

81

82 The M7.2 March 9th foreshock and the March 11th M7.1 aftershock have adequate

83 signal to noise ratio to be aligned and back-projected by the USarray (Fig.S3). The

84 two events appear to radiate coherent energies no longer than 30s with rupture length

85 around 30km. The two events also have different rupture directions, indicating no

86 systematic bias due to the array orientation. The deviation of the back-projection

87 locations of the initial phases with respect to their epicenter is due to imperfect

88 alignment of the waveforms. We use the M7.1 aftershock as the empirical Green's
89 function for the synthetic scenario tests in the next section.

90

91 **4. Synthetic tests of back-projection source imaging**

92

93 To understand the performance and potential bias of the back-projection we conduct
94 tests over a series of hypothetical earthquake scenarios. To include key characteristics
95 of the real wave field, such as the decay of the waveform coherence as a function of
96 time, we employ empirical Green's functions (eGf). Synthetics based on theoretical
97 Green's functions computed by ray theory and a spherically symmetric Earth model
98 tend to over-emphasize the effect of interference between multiple sources. We chose
99 the March 11th M7.1 aftershock as eGf event. The initial location error due to the
100 waveforms misalignment is corrected by shifting the initial HF radiator to the event
101 epicenter.

102 We first test a scenario of unilateral rupture to the South. We consider 10 uniform
103 sub-sources located every 15 km along the strike direction (Fig. S4). Their rupture
104 time corresponds to an assumed rupture speed of 1.5 km/s. The MUSIC
105 back-projection recovers the location and timing of the scenario sources very well.

106 The apparent secondary rupture to the West is an artifact due to the rupture pattern of
107 the eGf event itself.

108 We also tested more complicated scenarios inspired by our final results of the
109 back-projection of the Tohoku earthquake. Our second scenario is composed of five

110 different phases with various rupture speeds and directions (Fig. S5). The source
111 process is well recovered, although perturbed by modulation by the rupture pattern of
112 the eGf event. To minimize that effect, in our third scenario the inter-subevent
113 spacing is exaggerated (Fig. S6). Overall, the back projection results image our
114 synthetic scenarios adequately. In particular, the stages of different rupture speed are
115 reproduced. One of potential bias of the back projection imaging is that when rupture
116 temporally ceases, the coda of previous rupture would create apparent secondary
117 sources (Fig. S7). However, these sources show substantially smaller power than the
118 previously sources, which is unlikely to be the case of this earthquake, since we
119 observed sources of almost uniform power during the rupture.

120

121 **5. Array analysis of lower frequency recordings**

122

123 To explore the potential frequency-dependence of the spatial distribution of the source
124 radiation, we repeat the back-projection analysis of the Tohoku-Oki earthquake in two
125 lower frequency bands, 0.125-0.25 Hz and 0.25-0.5 Hz. Fig. S8 shows a map of the
126 peak energy radiators. Overall the lower frequency results follow the similar
127 down-dip distribution and temporal evolution as the high frequencies (Figure 1 of the
128 main text). The results from both arrays remain mutually consistent. However, the
129 lower frequency results are more scattered due to the expected lower resolution of the
130 array processing, which is inversely proportional to the wavelength.

131

132 **6. Analysis of strong motion data**

133

134 We examine velocity envelopes of strong motion data recorded along the
135 north-eastern Honshu coastline in order to identify prominent phases observed at
136 teleseismic distances. We bandpass filter the data, integrate, square and sum traces of
137 horizontal components. The resulting velocity envelopes are averaged using a sliding
138 median filter whose length is equal to five times the lower bound of the bandpass
139 filter. **Fig. S9a** and **Fig. S9b** show scaled smoothed velocity envelopes for 0.5-1 Hz
140 and 2.5-5 Hz frequency bands, respectively. We find that the prominent
141 high-frequency bursts from locations that ruptured 33, 85, and 140 seconds after the
142 mainshock are well resolved for frequencies as low as 0.5Hz. This observation is
143 consistent with our results for the 5-10 Hz frequency band presented in the main text.

144

145 **7. Velocity model**

146

147 In the analysis of the ground motion data recorded by the K-net and KiK-net seismic
148 networks we use the following 1D model derived from [Takahashi *et al.*, 2004]:

149

Depth (km)	S-wave velocity (km/s)
0	3.46
16	3.87

150

151

152 **7. Data sources**

153

154 We downloaded the data from the following sources:

155 USArray: <http://www.usarray.org/>

156 European seismic network: <http://www.orfeus-eu.org/>

157 K-net: <http://www.k-net.bosai.go.jp/>

158 Kik-net: <http://www.kik.bosai.go.jp/>

159 Italian network: <http://iside.rm.ingv.it/>

160

161

162 **References**

163

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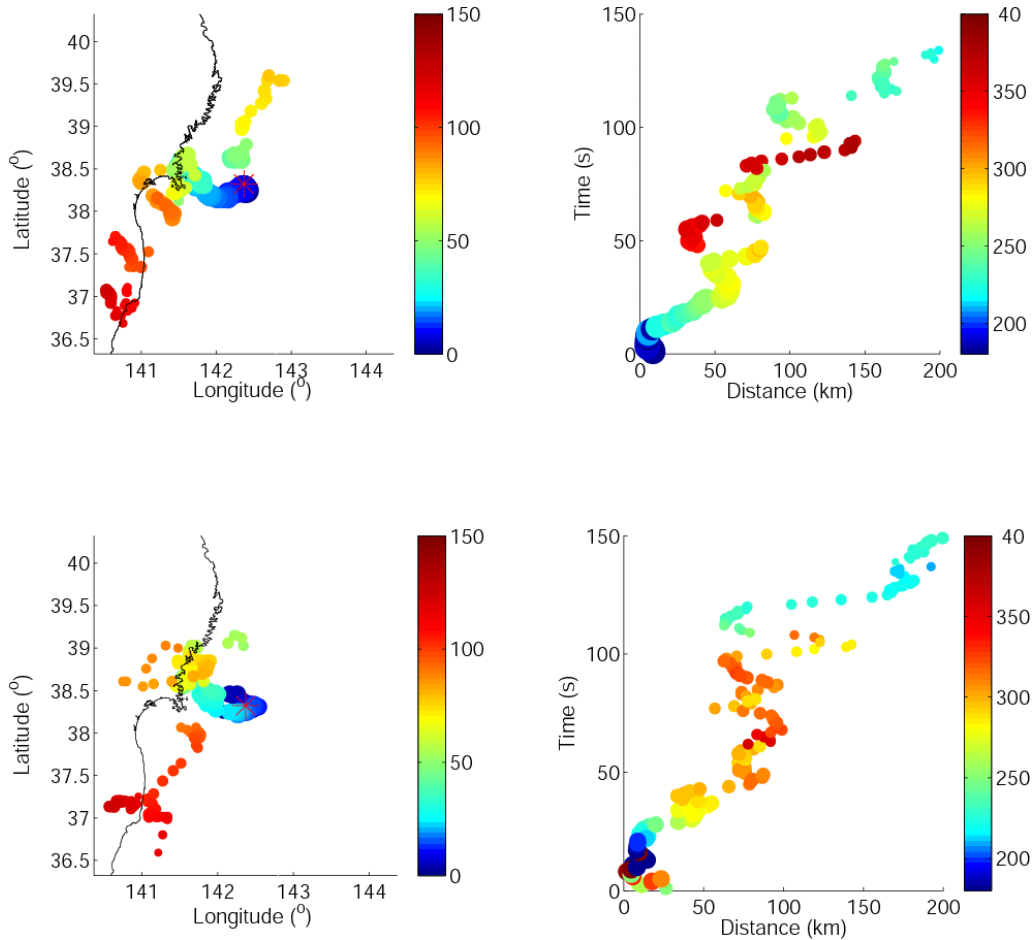
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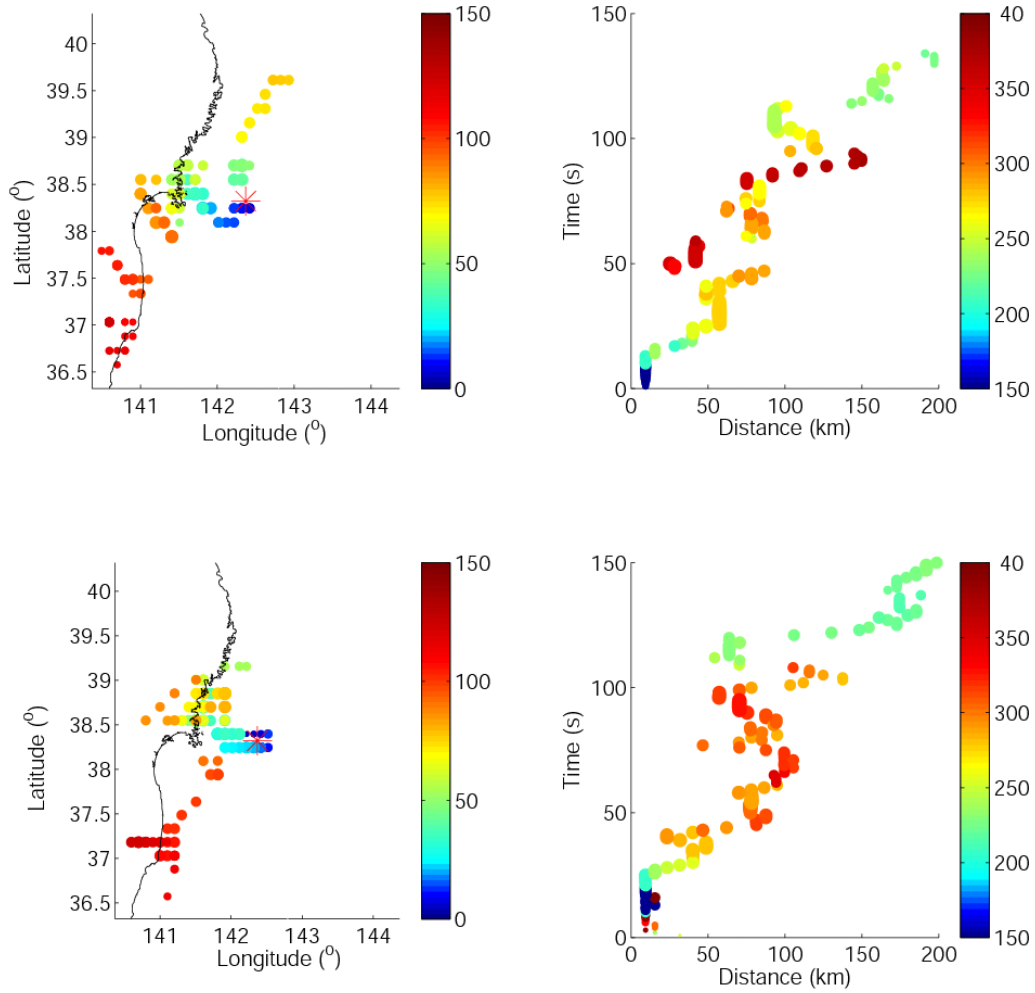
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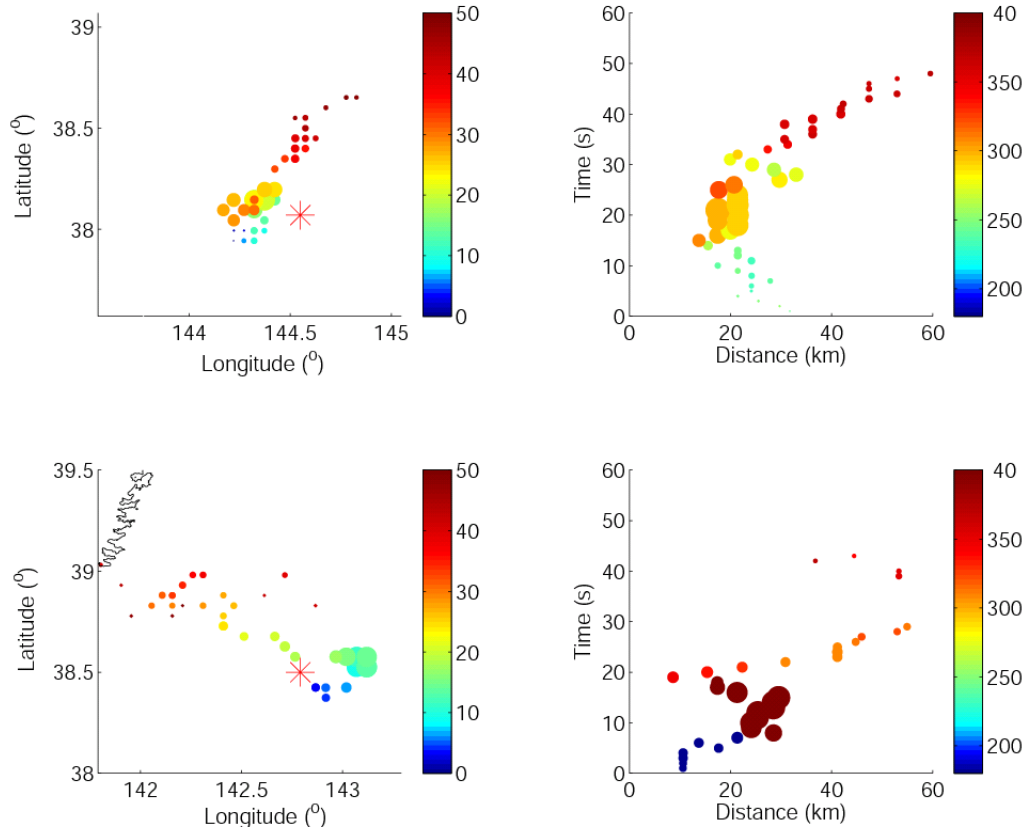
184

185 **Figure S1.** Back projections using correlation stacking at 0.5-1 Hz. *Left:* location of
 186 the strongest HF radiators color coded by rupture time. *Right:* rupture time versus
 187 epicentral distance, color coded by azimuth relative to the epicenter. *Top:* results
 188 based on European data. *Bottom:* results based on USarray data.



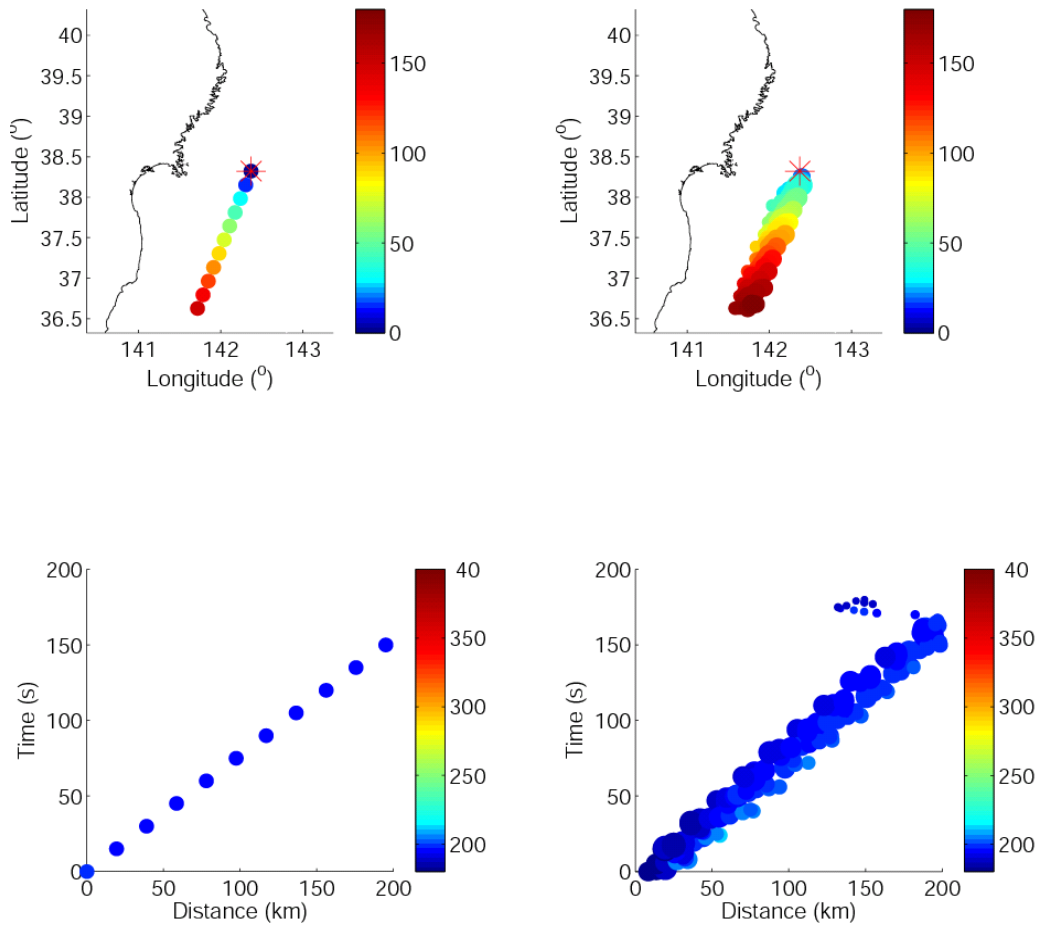
189

190 **Figure S2.** Back projections using cubic root stacking. *Left:* location of the strongest
 191 HF radiators color coded by rupture time. *Right:* rupture time versus epicentral
 192 distance, color coded by azimuth relative to the epicenter. *Top:* results based on
 193 European data. *Bottom:* results based on USarray data.



194

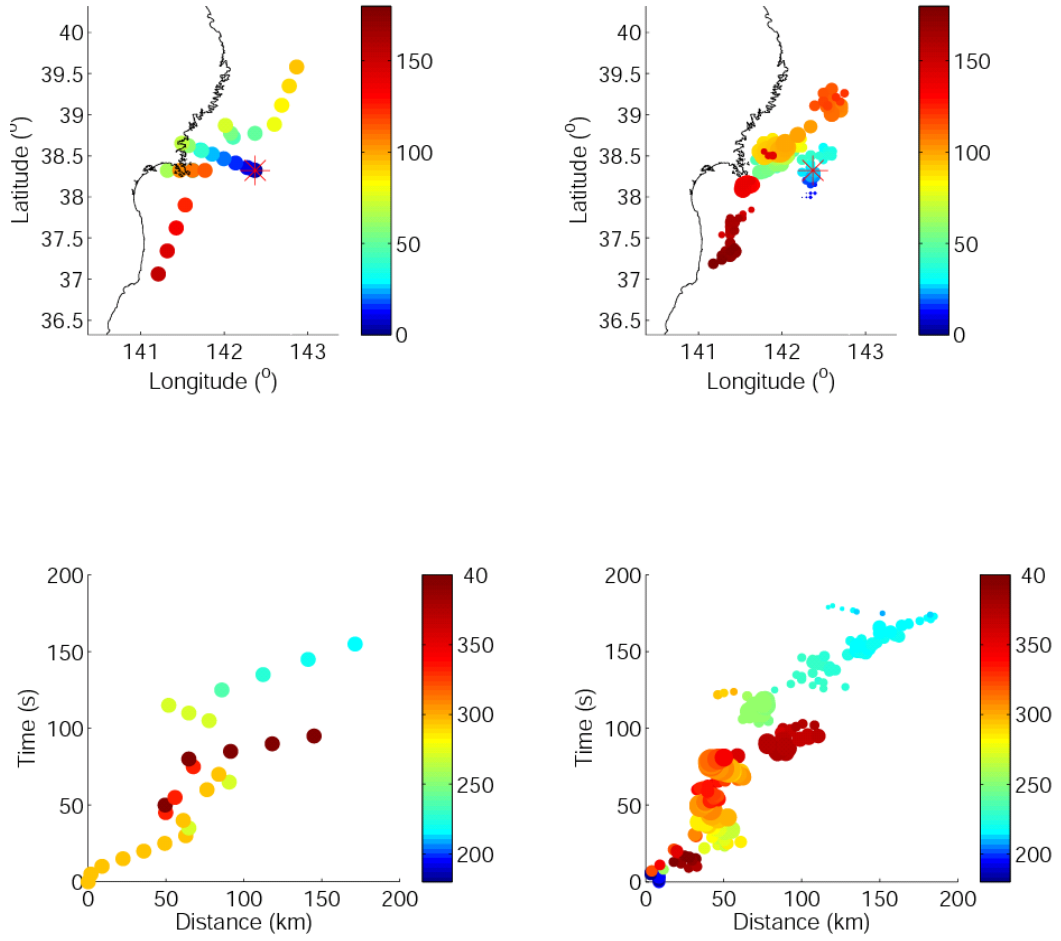
195 **Figure S3.** Foreshocks and aftershocks processed by the MUSIC technique at
 196 0.5-1Hz, based on USArray data. Same convention as as in figures S1 and S2. *Top:*
 197 the March 9th M7.2 foreshock. *Bottom:* the March 11th M7.1 aftershock.



198

199 **Figure S4.** Synthetic test 1. Same conventions as in previous figures. *Left:* synthetic

200 source scenario. *Right:* source model recovered by MUSIC back-projection.

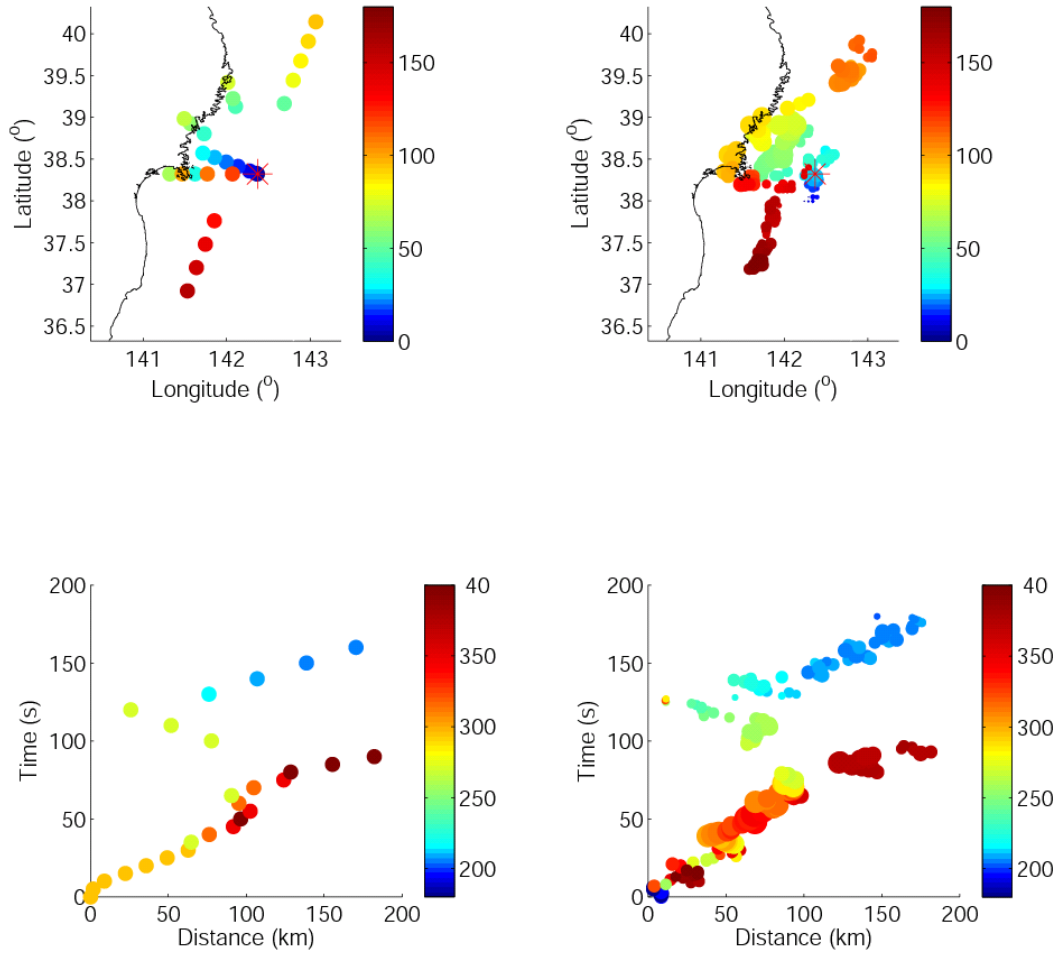


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202 **Figure S5.** Synthetic test 2. Same conventions as in previous figures. *Left:* synthetic

203 source scenario. *Right:* source model recovered by MUSIC back-projection.

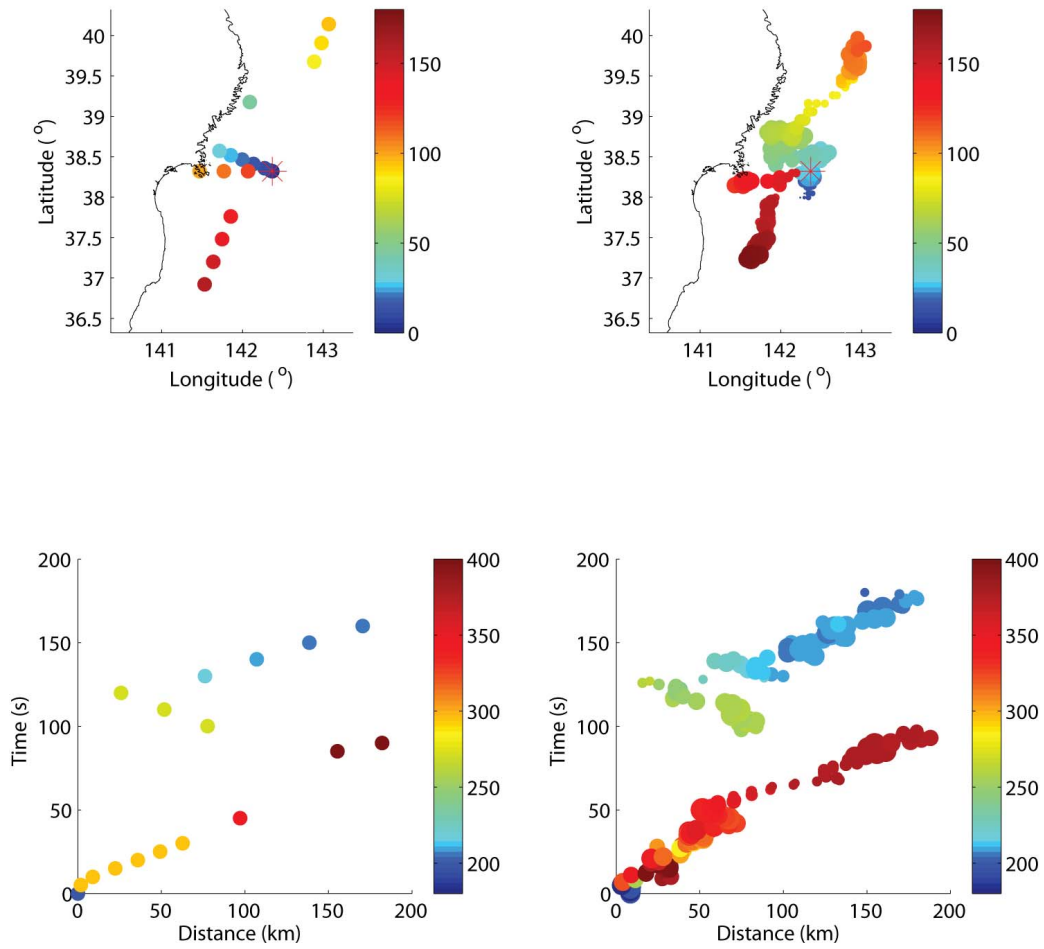
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206 **Figure S6.** Synthetic test 3. Same conventions as in previous figures. *Left:* synthetic

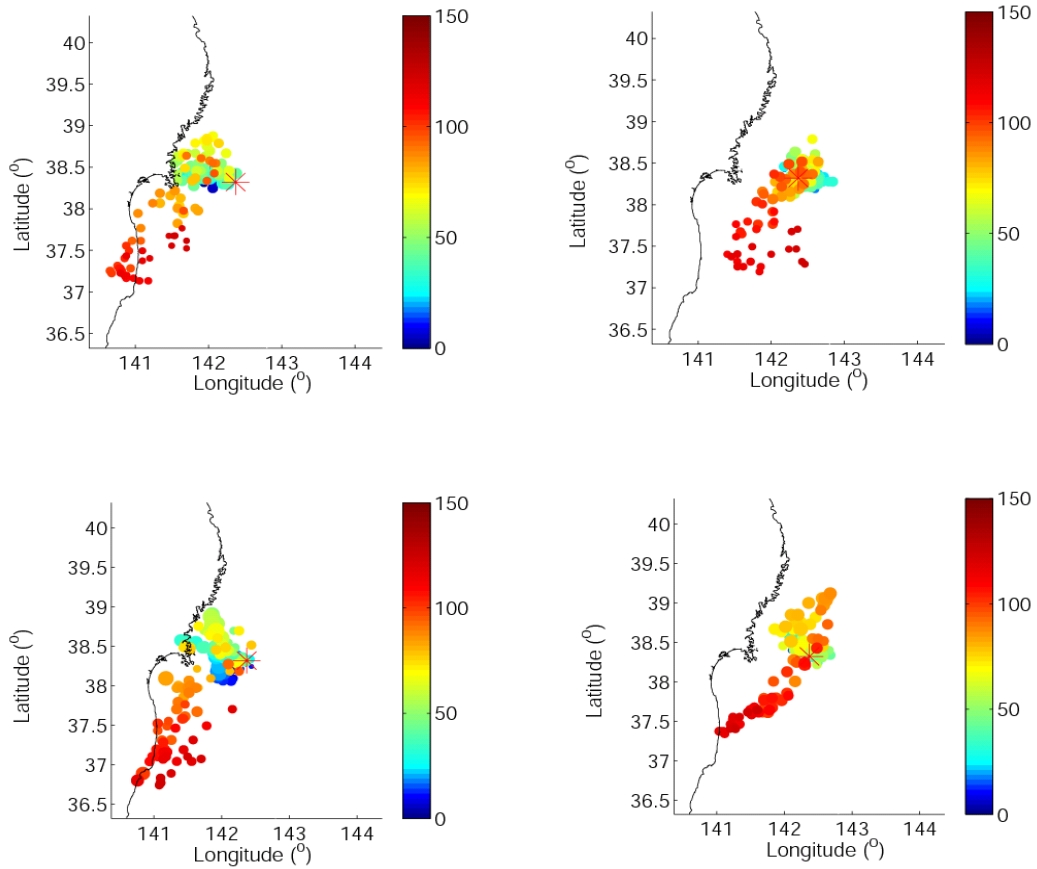
207 source scenario. *Right:* source model recovered by MUSIC back-projection.



208

209 **Figure S7.** Synthetic test 4. Same conventions as in previous figures. *Left:* synthetic

210 source scenario. *Right:* source model recovered by MUSIC back-projection.

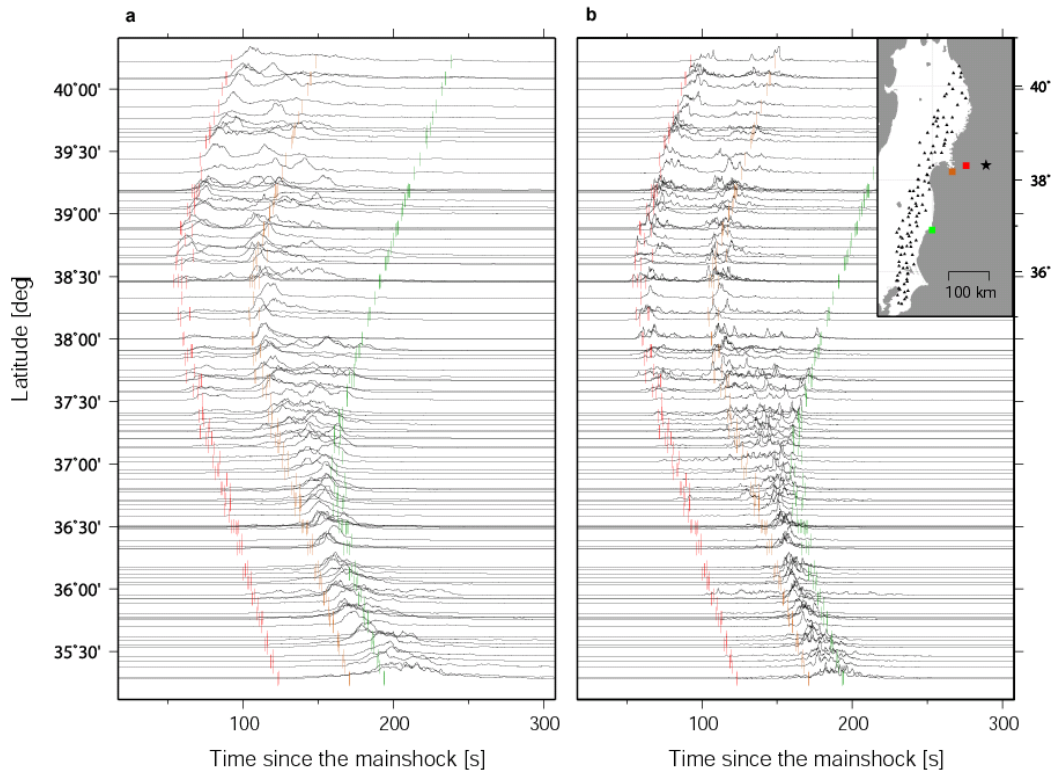


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212 **Figure S8.** Array analysis of lower frequency recordings. Location of the strongest

213 radiators color coded by rupture time. *Top:* 0.125 to 0.25 Hz. *Bottom:* 0.25 to 0.5 Hz.

214 *Left:* USarray. *Right:* European network.



215

216

Figure S9. Strong motion recordings at lower frequency band. Smoothed 0.5-1 Hz

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(left) and 2.5-5 Hz (right) velocity envelopes of strong motion data recorded along the

218

north-eastern Honshu coastline. Traces are scaled by their maximal amplitude. Star

219

and triangles in the inset map denote the epicentral location of the Tohoku Mw9.0

220

mainshock (USGS catalog reference) and the recording stations, respectively. Red,

221

yellow and green vertical bars along the time series indicate the the S-wave travel

222

times from asperities that ruptured 33s, 85s and 140s after that mainshock,

223

respectively. The asperities locations are indicated by squares of corresponding colors

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in the inset map.

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226