

1 **A mechanism for preseismic steady rupture fronts**
2 **observed in laboratory experiments**

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7 It has been shown that the onset of frictional instability is characterized
8 by a transition from stable, quasi-static rupture growth to unstable, inertially-
9 controlled high-speed rupture. In particular, slow rupture fronts propagat-
10 ing at a steady speed V_{slow} of the order of 5% of the S -wave speed have been
11 observed prior to the onset of dynamic rupture in recent fault-friction lab-
12 oratory experiments. However, the precise mechanism governing this V_{slow}
13 stage is unknown. Here we reproduce this phenomenon in numerical simu-
14 lations of earthquake sequences that incorporate laboratory-derived rate-and-
15 state friction laws. Our simulations show that the V_{slow} stage originates from
16 a stress concentration inherited from the coalescence of interseismic slow creep
17 fronts. Its occurrence is limited to a narrow range of the parameter space but
18 is found in simulations with two commonly-used state-variable evolution laws
19 in the rate-and-state formulation. The sensitivity of the speed V_{slow} to the
20 model parameters suggests that the propagation speed V_{slow} reported in lab-
21 oratory experiments may also be sensitive to parameters of friction and stress
22 conditions. Our results imply that time and space dimensions associated with
23 the propagation of V_{slow} on natural faults can be as much as a few seconds
24 and several hundred meters, respectively. Hence the detection of such pre-
25 seismic signals may be possible with near-field high-resolution observations.

1. Introduction

26 The onset of frictional instability is a key mechanism governing the nucleation of crustal
27 earthquakes and landslides. It has been demonstrated both in laboratory experiments
28 [e.g., *Dieterich*, 1979; *Ruina*, 1983; *Dieterich and Kilgore*, 1996; *Ohnaka and Shen*, 1999;
29 *Nielsen et al.*, 2010] and numerical simulations [e.g., *Okubo*, 1989; *Rice and Ben-Zion*,
30 1996; *Lapusta et al.*, 2000; *Rubin and Ampuero*, 2005] that the onset of frictional instabil-
31 ity is characterized by a transition from stable, quasi-static rupture growth to unstable,
32 inertially-controlled high-speed rupture. While these studies have advanced our under-
33 standing of the transitional behavior, direct comparisons between numerical models and
34 laboratory observations remain quite challenging due to difficulties in accurately moni-
35 toring the transitional behavior in laboratory friction experiments and in reproducing the
36 laboratory observations in numerical simulations that incorporate appropriate friction
37 laws.

38 Several laboratory studies attempted to understand the behavior of the quasi-static
39 to dynamic transition of stick-slip motion on experimental faults (i.e., pre-cut interfaces).
40 *Ohnaka and Shen* [1999] reported an initial, quasi-static phase in which the rupture grows
41 at a slow and steady speed, followed by a rupture acceleration phase up to dynamic
42 speeds. In friction experiments on polymethyl-methacrylate (PMMA), *Rubinstein et al.*
43 [2004] observed a slow detachment front propagating at 5% of the S -wave speed (V_s) after
44 the passage of a supershear rupture front. More recently, *Nielsen et al.* [2010] reported
45 steady rupture fronts systematically propagating at about 5% of V_s on experimental faults
46 where a stick-slip instability was spontaneously nucleated under slow loading. However,

47 under different loading conditions, *Ben-David et al.* [2010] found variability of rupture
 48 speeds ranging from a few percent of V_s to P -wave speed (V_p) depending on the ratio of
 49 local shear to normal stresses, suggesting that the speed of slow steady fronts ($\sim 5\%$ of
 50 V_s) observed in *Rubinstein et al.* [2004] and *Nielsen et al.* [2010] is not universal. Hence
 51 the mechanism of a slow steady propagation front remains elusive.

52 In this study, we reproduce the occurrence of slow steady propagation fronts preceding
 53 fast dynamic rupture in numerical simulations of spontaneous earthquake sequences, con-
 54 sistent with the laboratory findings of *Nielsen et al.* [2010]. We refer to this phenomenon
 55 as a ‘ V_{slow} stage’ and to its propagation speed as ‘ V_{slow} ’. We discuss the mechanism of the
 56 V_{slow} stage, the dependence of V_{slow} on model parameters, and its potential occurrence on
 57 natural faults.

2. Model set-up

58 We conduct 2-D simulations of spontaneous earthquake sequences on a 1-D fault sub-
 59 jected to slow, tectonic loading [*Kaneko et al.*, 2010]. We consider an antiplane (Mode
 60 III) configuration in which purely dip-slip motion is assumed. The simulations resolve
 61 all stages of the seismic cycle: the aseismic nucleation process, the subsequent dynamic
 62 rupture event, the postseismic slip, and the interseismic quasi-static deformation between
 63 events.

64 The fault resistance to sliding is described by laboratory-derived rate and state friction
 65 laws [*Dieterich*, 1979; *Rice and Ruina*, 1983; *Ruina*, 1983]. For time-independent effective
 66 normal stress $\bar{\sigma}$, the shear strength τ on the fault is expressed as

$$\tau = \bar{\sigma} \left[f_0 + a \ln(\dot{\delta}/\dot{\delta}_0) + b \ln(\dot{\delta}_0 \theta / L) \right], \quad (1)$$

67 where a and b are rate and state constitutive parameters, $\dot{\delta}$ is slip velocity, f_0 is a reference
 68 friction coefficient corresponding to a reference slip velocity $\dot{\delta}_0$, θ is a state variable, and
 69 L is the characteristic slip for state evolution [e.g., *Dieterich*, 1979; *Ruina*, 1983; *Rice and*
 70 *Ruina*, 1983]. We consider the two most classical state variable evolution laws:

$$d\theta/dt = 1 - \dot{\delta}\theta/L \quad (\text{aging law}) , \quad (2)$$

$$d\theta/dt = -(\dot{\delta}\theta/L) \ln(\dot{\delta}\theta/L) \quad (\text{slip law}) \quad (3)$$

71 [*Dieterich*, 1979; *Ruina*, 1983; *Rice and Ruina*, 1983]. Recently, *Ampuero and Rubin*
 72 [2008] rekindled the discussion of which state evolution laws are more appropriate to use
 73 in earthquake modeling. *Bayart et al.* [2006] showed that the slip law provides a better
 74 match to velocity-jump experiments. On the other hand, *Beeler et al.* [1994] found, during
 75 slide-hold-slide experiments using a servo-control system, that a frictional surface heals
 76 with time, rather than with slip rate, suggesting that the aging law is a more appropriate
 77 representation.

78 The parameter combination $a - b < 0$ corresponds to steady-state velocity-weakening
 79 friction and can lead to unstable slip, whereas $a - b > 0$ corresponds to steady-state
 80 velocity-strengthening and leads to stable sliding. Throughout this article, we omit the
 81 words “steady-state” and simply refer to velocity weakening or strengthening.

82 Under slow tectonic loading, a frictional instability (i.e., an earthquake) is able to
 83 develop only if the velocity-weakening region of the fault exceeds the nucleation size h^*
 84 [*Rice and Ruina*, 1983; *Rubin and Ampuero*, 2005]. Two theoretical estimates of the
 85 earthquake nucleation size for 2-D problems are given by

$$h_{\text{RR}}^* = \frac{\pi}{4} \frac{\mu L}{\bar{\sigma}(b - a)} , \quad (4)$$

$$h_{\text{RA}}^* = \frac{2}{\pi} \frac{\mu L b}{\bar{\sigma} (b - a)^2}, \quad (5)$$

86 where μ is shear modulus for mode III. The estimate h_{RR}^* was derived from the linear
 87 stability analysis of steady sliding by *Rice and Ruina* [1983], while h_{RA}^* was obtained
 88 for $a/b \gtrsim 0.5$ by *Rubin and Ampuero* [2005] on the basis of energy balance for a quasi-
 89 statically expanding crack governed by the aging law (2). *Rubin and Ampuero* [2005] gave
 90 formulae for half of the nucleation size but we use full sizes.

91 Our simulated fault is 300-cm long. Slip evolution is computed based on the assumed
 92 friction law on the 150-cm long central portion. The slip rate $\dot{\delta}_{\text{Load}} = 5$ cm/year is
 93 prescribed on the two 75-cm long outer portions of the fault. The central portion is
 94 divided into three segments: a 75-cm long velocity-weakening segment surrounded by
 95 two 37.5-cm long velocity-strengthening segments (Fig. 1a). The velocity-strengthening
 96 segments mimic viscous silicon patches placed at each end of the fault in some of the
 97 experiments of *Nielsen et al.* [2010], creating similar loading conditions. Note that, in
 98 the study of *Nielsen et al.* [2010], the V_{slow} stage was observed for smooth fault surfaces
 99 with or without viscous patches (S. Nielsen, written communication, 2009), but was not
 100 observed for rough surfaces with viscous patches.

101 Table 1 gives a range of model parameters considered in this study. Since values of rate-
 102 and-state parameters a , b , and L are not known for the laboratory specimen of *Nielsen*
 103 *et al.* [2010], we do not attempt to exactly reproduce the spatial and temporal scales of the
 104 V_{slow} stage observed in their laboratory experiments. Instead, we set the spatial scale and
 105 nucleation sizes to be roughly equal to those in the laboratory experiments by adjusting
 106 the value of L . The values of other model parameters are chosen to represent plausible

107 stress and friction values on natural faults in seismogenic conditions rather than those in
108 the laboratory experiments.

3. Simulations of slow steady propagation fronts

109 Fig. 1 shows one of the simulation examples in which slow steady fronts (V_{slow}) propa-
110 gate at about 5% of V_s systematically during the nucleation processes of seismic events.
111 Motivated by the fact that a/b of many velocity-weakening materials in laboratory exper-
112 iments is closer to 1 than 0, we set $a/b = 0.93$. In this example (Fig. 1b) we adopt the
113 slip law (3). The blue lines show the continuous slow sliding of the velocity-strengthening
114 segments, which creates a stress concentration at its tip and penetrates into the velocity-
115 weakening segment (Fig. 1b). In due time, seismic rupture nucleates and propagates
116 bilaterally (its progression is shown by red lines in Fig. 1b). After a seismic event,
117 the velocity-strengthening segments experience postseismic sliding due to the transferred
118 stress. The interseismic period between two successive events is 5 hours.

119 Fig. 1c shows a close-up look at the onset of one of the seismic events. The rupture
120 fronts, the positions of which are defined here by the peak values of shear stress, begin
121 at a localized point, expand quasi-statically, then steadily propagate at about 5% of V_s
122 before accelerating to much faster speeds. The behavior of the V_{slow} fronts (Fig. 1c) is
123 similar to that in the laboratory experiments of *Nielsen et al.* [2010] (Fig. 1d) in that
124 the speed V_{slow} is about 5% of V_s and remains about the same for all events in a given
125 simulation (except for the first few that are affected by initial conditions).

126 To understand the mechanism of the slow steady fronts, we look at the evolution of
127 slip rates $\dot{\delta}$, shear stress, and the quantity $\dot{\delta}\theta/L$ before and during the occurrence of the

128 V_{slow} stage (Fig. 2). During the interseismic period, two creep fronts emanate from the
 129 rheological boundaries ($x \approx 45, 105$ cm) and then propagate inward. Their eventual
 130 coalescence creates favorable conditions for earthquake nucleation near the center of the
 131 fault (Figs. 2a,c,e). Behind the two creep fronts, the condition is near steady-state
 132 ($\dot{\delta}\theta/L \approx 1$), and there is no evolution of state variable θ according to (3). The V_{slow} stage
 133 originates from a stress concentration inherited from the coalescence of the creep fronts
 134 (Figs. 2b,d,f). During the V_{slow} stage, the peak slip velocities remain roughly constant in
 135 time (~ 0.05 m/s in this example). The expansion of rupture takes the form of a bilateral
 136 crack (Fig. 2b) as opposed to a unilateral pulse reported by *Ampuero and Rubin* [2008]
 137 under different loading conditions.

138 The relation between propagation speed V_{slow} and peak slip rate $\dot{\delta}_{\text{max}}$ is consistent with
 139 a theoretical relation given by equation (53) of *Ampuero and Rubin* [2008]:

$$V_{\text{prop}} \approx 0.75 \frac{\mu \dot{\delta}_{\text{max}}}{b\bar{\sigma}} \left(\ln \frac{\dot{\delta}_{\text{max}} \theta_i}{L} \right)^{-1}, \quad (6)$$

140 where θ_i is the value of the state variable prior to the arrival of the rupture front. For
 141 instance, for the case shown in Fig. 2, $\ln(\dot{\delta}_{\text{max}} \theta_i / L) \approx 11-16$ and $\dot{\delta}_{\text{max}} \sim 0.05$ m/s, which
 142 yields $V_{\text{prop}}/V_s = 0.04-0.06$. This is consistent with the speed V_{slow} obtained in the simu-
 143 lation (Fig. 1c).

144 Setting $\dot{\delta}_{\text{max}} = 2a\sigma V_s/\mu$ in equation (6), the slip velocity at which the effect of radia-
 145 tion damping is comparable to the direct effect of rate-and-state friction, *Perfettini and*
 146 *Ampuero* [2008] proposed that a typical rupture speed at the onset of elastodynamic ef-
 147 fects was $V_{\text{prop}} \sim 0.05V_s$. To assess the relevance of elastodynamics, Fig. 2 shows results
 148 obtained by first simulating three earthquake cycles (Fig. 1b), then continuing with a

149 quasi-static simulation (by turning off the inertial effects). We find that the V_{slow} stage
 150 occurs despite the quasi-static assumption, suggesting that the speed V_{slow} does not ex-
 151 plicitly depend on V_s . A subtle effect of elastodynamics is however not discarded because
 152 the arrest of the last dynamic event sets the background conditions for the creep front
 153 propagation.

154 A fracture mechanics argument based on an idealized model provides insight on how the
 155 stress concentration induced by the coalescence of the creep fronts gives rise to a period of
 156 a steady propagation speed. Let us consider the quasi-static growth of a mode III crack
 157 of length 2ℓ under a stress field that consists of a uniform background value (and hence
 158 constant stress drop $\overline{\Delta\tau}$) plus a highly concentrated force F at the center of the crack.
 159 The balance between the energy release rate at the crack tip, G , and the fracture energy,
 160 G_c , provides a relation between crack speed V_{prop} and crack half-length ℓ . On the one
 161 hand, the static energy release rate G , given by [e.g., *Ampuero et al.*, 2006]

$$G(\ell) = \frac{\pi\ell}{2\mu} \left(\overline{\Delta\tau} + \frac{2F}{\pi\ell} \right)^2, \quad (7)$$

162 reaches a minimum when $\ell = \ell_{\text{min}} = 2F/(\pi\overline{\Delta\tau})$. On the other hand, the effective fracture
 163 energy of rate-and-state friction behaves as $G_c \approx b\bar{\sigma}L \ln(\dot{\delta}_{\text{max}}/\dot{\delta}_{\text{Load}})^n$, where $n = 1$ for the
 164 slip law and $n = 2$ for the aging law [*Ampuero and Rubin*, 2008]. Equation (6) implies
 165 that $\dot{\delta}_{\text{max}}$, and hence G_c , is an increasing function of V_{prop} . Since $G(\ell) = G_c(V_{\text{prop}})$, the
 166 crack propagation speed is almost steady when the energy release rate $G(\ell)$ is almost
 167 constant. This happens when the crack length is close to $2\ell_{\text{min}}$. This analysis shows that
 168 a quasi-statically expanding crack subjected to a stress concentration can lead to a period
 169 of a steady propagation speed. The previous analysis also provides an estimate of the

170 propagation speed, which depends on the product $F\Delta\tau$. However, deriving an adequate
 171 estimate of F is not straightforward, and the quantitative comparison of the idealized
 172 model to the numerical simulation results remains a subject of future work.

4. Dependence of the speed of steady propagation fronts on model parameters

173 To identify the parameters controlling V_{slow} , we perform a number of earthquake-
 174 sequence simulations with different sets of model parameters indicated in Table 1. First,
 175 we vary the size of the velocity-weakening segment D_{vw} while other parameters are held
 176 fixed. We normalize D_{vw} by $h_{\text{RR}}^*/2$, a length that agrees well with our simulated nucle-
 177 ation sizes. We find that by increasing D_{vw}/h^* the speed V_{slow} increases (Fig. 3a), the
 178 duration of the V_{slow} stage decreases dramatically, and its spatial extent remains about
 179 the same (Fig. 3b).

180 Different speeds of V_{slow} fronts (Fig. 3a) are related to different amplitudes of slow creep
 181 fronts during the interseismic period. For a larger value of D_{vw}/h^* , the slip rate of slow
 182 creep fronts is larger (e.g., $\sim 10^{-8}$ vs. $\sim 10^{-9}$ m/s in Fig. 3c) and, upon their coalescence,
 183 the resulting peak slip rate ($\dot{\delta}_{\text{max}}$) during the V_{slow} stage also becomes larger (Fig. 3c).
 184 From (6), the speed V_{slow} is strongly correlated to the peak slip rate (Fig. 3b). Hence the
 185 coalescence of larger-amplitude creep fronts emerging during the interseismic period leads
 186 eventually to a faster propagation speed during the V_{slow} stage.

187 We also vary the value of the rate-and-state parameter b in the velocity-weakening
 188 segment. We find that V_{slow} decreases with increasing a/b (Fig. 3d). This behavior can
 189 be attributed to the dependence of V_{slow} on D_{vw}/h^* shown in Fig. 3a, which indicates that
 190 a smaller nucleation size h^* leads to a faster V_{slow} ; theoretical estimates (4) and (5) and

191 numerical simulations show that h^* generally decreases as b increases. When a/b becomes
 192 even smaller ($a/b < 0.88$), the nucleation occurs near one of the rheological transitions
 193 and takes the form of a unilateral pulse (Figs. 4a,d). Since the nucleation size becomes
 194 smaller and the fault becomes more unstable with larger $b - a$, one of the slow creep fronts
 195 can self-accelerate towards earthquake nucleation, without coalescence (Fig. 4d). Its peak
 196 slip rate increases monotonically with time, and so does its propagation speed as predicted
 197 by (6) (Figs. 4a,d). Hence there is no steady V_{slow} stage when the nucleation occurs near
 198 one of the rheological transitions (Figs. 4a,d). This behavior was also observed by *Nielsen*
 199 *et al.* [2010] in experiments with rough surfaces and viscous patches. We find that this
 200 style of nucleation is the most common in the parameter range shown in Table 1. This is
 201 probably why a V_{slow} stage was not reported in previous theoretical studies.

202 Since the assumed loading rate $\dot{\delta}_{\text{Load}}$ is orders of magnitudes slower than that in the
 203 laboratory experiments, we further explore the dependence of V_{slow} on $\dot{\delta}_{\text{Load}}$. Fig. 3e
 204 shows that the speed V_{slow} increases with the loading rate. *Kaneko and Lapusta* [2008]
 205 demonstrated how increased loading conditions could change the nucleation process and,
 206 in particular, cause order-of-magnitude smaller nucleation sizes. Since a larger loading
 207 rate leads to a smaller nucleation size, the quantity D_{vw}/h^* increases with the loading
 208 rate, resulting in larger V_{slow} in our simulations (Fig. 3e).

209 We also find the occurrence of a V_{slow} stage in simulations with the aging law (2)
 210 (Fig. 4c). The dependence of the properties of the V_{slow} stage on model parameters is
 211 qualitatively similar to the case of the slip law, although the parameter ranges in which a
 212 V_{slow} stage occurs are different. Under the same rate and state parameters, $V_{\text{slow}} \sim 0.05V_s$

213 when $D_{vw}/h^* \sim 24$ for the slip law and $D_{vw}/h^* \sim 3.4$ for the aging law, respectively
214 (Figs. 4a,b). For both laws a V_{slow} stage exists when nucleation proceeds by coalescence
215 of two slow creeping fronts (Figs. 4c,f), but not if nucleation occurs near a rheological
216 transition (Figs. 4b,e).

5. Discussion

217 We have found that in rupture models nucleated by the coalescence of aseismic slip
218 fronts, dynamic rupture is preceded by a stage of slow, steady rupture propagation. Its
219 rupture speed V_{slow} is not universal, but depends on frictional and loading parameters
220 (Figs. 3 and 4). This suggests that the value $V_{\text{slow}} \sim 0.05V_s$ reported in the laboratory
221 experiments of *Nielsen et al.* [2010] may also be sensitive to parameters of friction and
222 stress conditions. Future work may be directed towards determining the values of the
223 rate-and-state parameters of the laboratory specimen to allow quantitative comparison
224 between laboratory observations and numerical simulations.

225 Slow propagation fronts have been observed in laboratory experiments under a vari-
226 ety of loading conditions [e.g., *Rubinstein et al.*, 2004; *Nielsen et al.*, 2010; *Ben-David*
227 *et al.*, 2010]. In this study, the occurrence of a V_{slow} stage results from coalescence of
228 slow creep fronts emanating from rheological boundaries, and hence loading conditions
229 play an important role in generating a V_{slow} stage. We have not yet explored scenarios
230 where the fault is characterized by velocity-weakening conditions only (i.e., no rheological
231 transitions). Whether the V_{slow} stage would occur under such scenarios remains a subject
232 of future work.

233 Earthquake nucleation is relevant to earthquake prediction because nucleation deter-
234 mines the origin time and hypocenter of seismic rupture and may result in detectable
235 precursors. Laboratory rock-sliding experiments showed that values of characteristic slip
236 L range from ~ 1 to ~ 500 μm , depending on the fault roughness and gouge width [Marone,
237 1998]. Space dimension in numerical models can be non-dimensionalized, for example, by
238 $x' = x/L$ such that the results of numerical simulations can be scaled with L . The results
239 shown in Figs. 1c and 4c combined with the upper bound of the laboratory value $L = 500$
240 μm suggest that the space and time dimensions associated with hypothetical V_{slow} fronts
241 propagating at 5% of V_s on natural faults correspond to ~ 100 m and ~ 0.6 s (= 100 m
242 divided by $0.05V_s$) for the cases with the slip law, respectively, and ~ 500 m and ~ 3 s (=
243 500 m divided by $0.05V_s$) for the cases with the aging law, respectively. This implies that
244 inferences of such preseismic steady rupture propagation before the break-out of a seismic
245 event may be possible with near-field high-resolution observations.

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Table 1. A range of parameters used in this study.

Parameter	Symbol	Slip-law cases	Aging-law cases
Shear modulus	μ	32.0 GPa	32.0 GPa
Shear wave speed	V_s	3.464 km/s	3.464 km/s
Reference slip rate	$\dot{\delta}_0$	10^{-6} m/s	10^{-6} m/s
Reference friction coefficient	f_0	0.6	0.6
Characteristic slip	L	0.1 microns	0.1 microns
Effective normal stress	$\bar{\sigma}$	50 MPa	50 MPa
Rate and state parameter a	a	0.0100	0.0100
Rate and state parameter b	b	0.0105-0.0140 ^a	0.0108-0.0140 ^a
Size of velocity-weakening segment	D_{vw}	50-112 cm	56-100 cm
Loading rate	$\dot{\delta}_{Load}$	1-100 cm/yr	5 cm/yr

^a The indicated values of b are valid for the VW region.

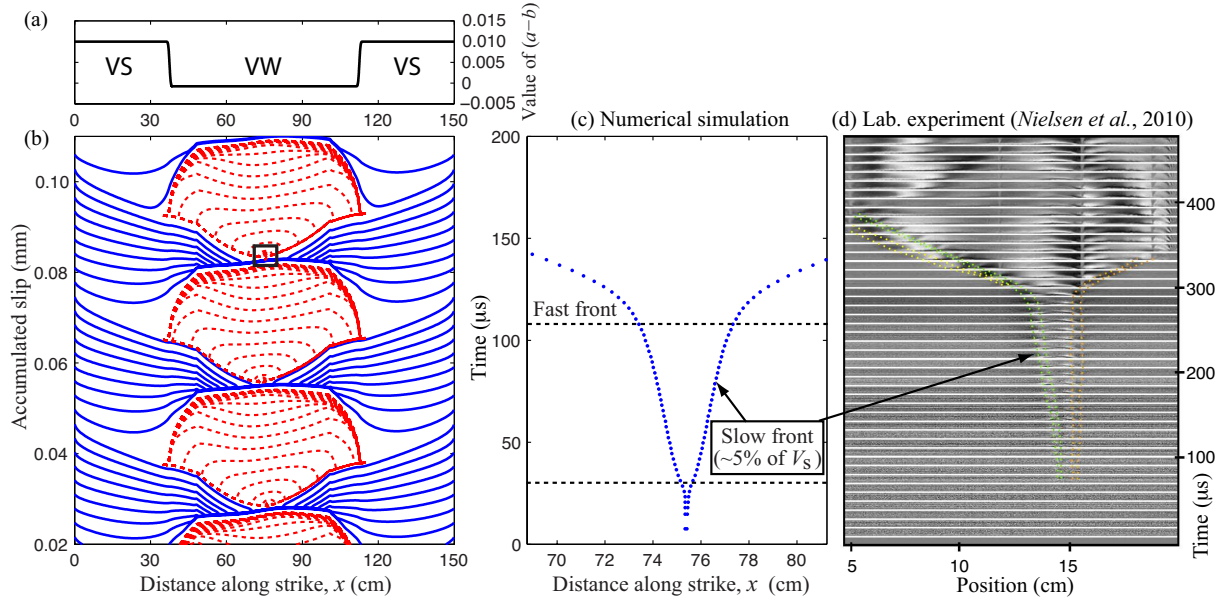


Figure 1. An example of preseismic slow-propagation fronts ($\sim 5\%$ of V_s) in long-term fault-slip simulations. (a) The assigned distribution of the friction parameter ($a - b$) with $a = 0.0100$, $b = 0.0108$, and $a/b = 0.93$ in the velocity-weakening (VW) segment and $a = 0.0100$ and $b = 0$ in the velocity-strengthening (VS) segments. (b) The assigned distribution of the friction parameter results in a sequence of shear-slip events similar to that in the laboratory experiments of *Nielsen et al.* [2010]. Red and blue curves display slip accumulation every $25 \mu\text{s}$ during the simulated earthquakes and every 0.55 hours, respectively. The black square indicates a location where one of the seismic events nucleates. (c) Positions of the rupture fronts as a function of time during a transition from quasi-static to dynamic rupture. Time $t = 0$ is chosen sometime before the V_{slow} stage. The rupture fronts, defined as the locations of peak shear stresses, propagates bilaterally with two distinct speeds: $V_r \sim 0.05V_s$ (slow front) and $V_r \gg 0.05V_s$ (fast front). (d) A sequence of interferometric photograms showing the nucleation and the propagation of rupture fronts in the laboratory experiments of *Nielsen et al.* [2010].

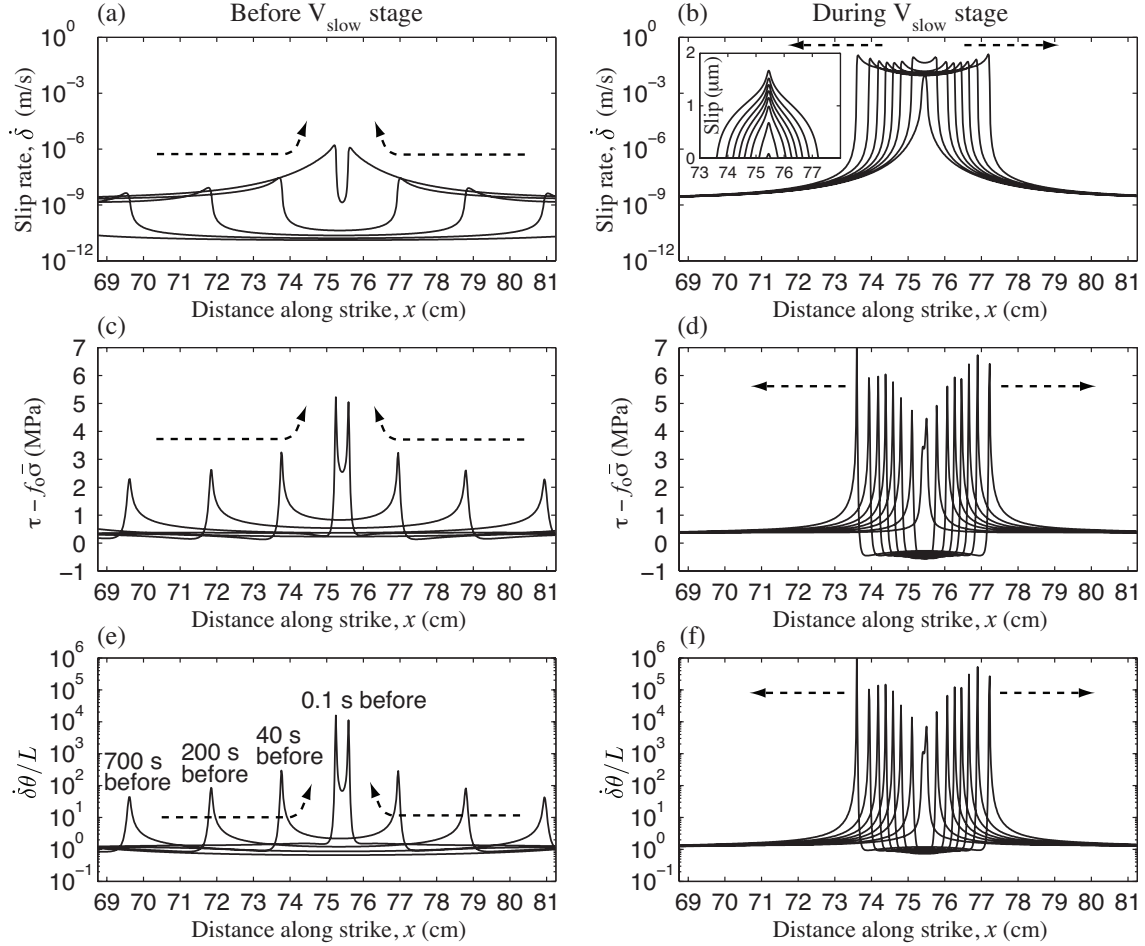


Figure 2. Snapshots of (a, b) slip rates, (c, d) shear stress τ with respect to a reference stress value $f_0 \bar{\sigma}$, and (e, f) $\delta\theta/L$, before and during the V_{slow} stage. Each snapshot in (a, c, e) is taken at times before the onset of the V_{slow} stage indicated in (e). Snapshots in (b, d, f) are taken every $13 \mu\text{s}$. Arrows indicate the propagation direction of the peak values in each plot. The inset in (b) shows slip accumulation during the V_{slow} stage, indicating a crack-like expansion of rupture growth. The propagation of V_{slow} originates from a stress concentration inherited from the coalescence of the slow creep fronts. In this figure, the result from a quasi-static simulation is shown, and the outermost curves in (b, d, f) correspond to the time just before the quasi-static solution ceased to exist.

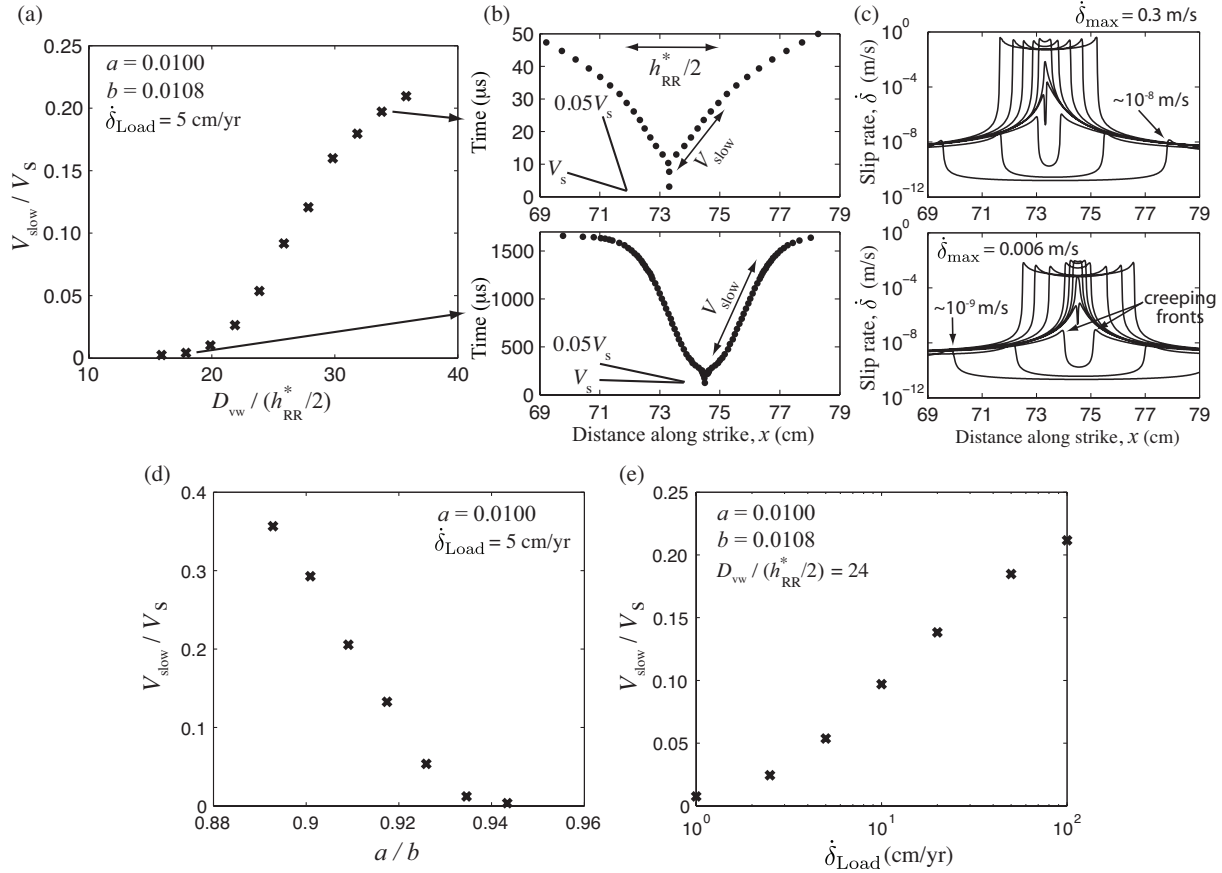


Figure 3. Dependence of the speed V_{slow} of slow propagation fronts on model parameters. (a) V_{slow}/V_s vs. the size of the velocity-weakening segment D_{vw} divided by a theoretical estimate of the nucleation size $h_{\text{RR}}^*/2$, with $h_{\text{RR}}^*/2 = 3.1 \text{ cm}$ for the parameters used. V_{slow} increases with D_{vw}/h^* . The evolution of (b) peak shear stress and (c) slip rates for two end-member cases shown in (a). Top insets of (b, c) correspond to the same case. The peak slip rates during the V_{slow} stage and of the slow creeping fronts during the interseismic period are an order of magnitude larger for the case with larger V_{slow} . (d) V_{slow}/V_s vs. a/b in the velocity-weakening segment, with $a = 0.01$. V_{slow} decreases as a/b increases. (e) V_{slow}/V_s vs. the loading rate $\dot{\delta}_{\text{Load}}$. Note that the horizontal axis is on a logarithmic scale. V_{slow} increases with the loading rate. Indicated parameters and their values are held unchanged for the results shown in (a, d, e).

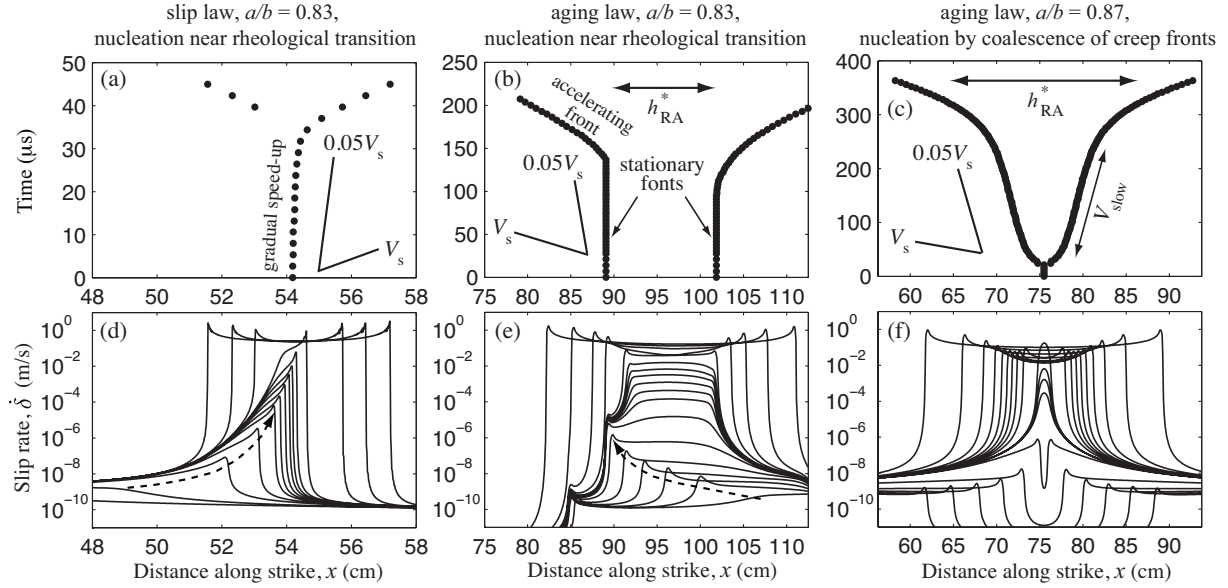


Figure 4. Nucleation processes for the cases with (a, d) the slip law and $a/b = 0.83$, (b, e) the aging law and $a/b = 0.83$, and (c, f) the aging law and $a/b = 0.87$. The evolution of (a-c) peak shear stress and (d-f) slip rates in each case is shown. A theoretical estimate of the nucleation size h_{RA}^* given in (5) is indicated for the cases with the aging law. In (d, e), the nucleation occurs near the rheological transition where a slow creep front becomes unstable. Arrows in (d, e) indicate the propagation direction of the slow creep front. In (f), the coalescence of two slow creep fronts during the interseismic period leads to a nucleation characterized by the V_{slow} stage prior to the onset of the unstable rupture.