

For problem sets 4 and 5, we will construct a simple box model to study the carbon cycle. In this problem, we'll begin with constructing modules for ocean uptake and terrestrial uptake of atmospheric CO₂. For this problem set, your model will have three boxes: an atmospheric box, a surface ocean box, and a terrestrial biosphere box. Fluxes of CO₂ to the atmosphere will be positive, and fluxes to the ocean or terrestrial biosphere will be negative. You should keep track of moles of CO₂ rather than mass.

(1) What is the equilibrium concentration of CO₂ in sea water if the atmospheric concentration is at its preindustrial level of 280 ppm? The Henry's Law constant at 24 deg is 0.0284 mol (kg-soln)⁻¹ atm⁻¹. We will ignore salinity effects in this model. What is the equilibrium concentration of CO₂ in sea water if atmospheric CO₂ doubles? Assume that the concentration of CO₂ doubles instantaneously - what is the e-folding response time to achieve a new equilibrium assuming the CO₂ diffuses into the ocean at 24 deg C? What is the concentration of dissolved CO₂ when this occurs? What is the equilibrium concentration after CO₂ has doubled?

(2) Terrestrial gross primary productivity (GPP) can be parameterized as:

$$GPP = \lambda \cdot EVI \cdot T_{scale} \cdot W_{scale} \cdot L_{scale} \quad (1)$$

In this equation, λ gives the maximum uptake rate for CO₂ and is dependent on ecosystem type. EVI is a proxy for density of vegetation. T_{scale} and W_{scale} modify the rate of GPP when temperature and moisture conditions change. The last term modifies the rate of GPP based on photosynthetically active radiation (PAR) received by plants.

$$T_{scale} = \frac{(T - T_{min}) \cdot (T - T_{max})}{(T - T_{min}) \cdot (T - T_{max}) - (T - T_{opt})^2} \quad (2)$$

$$W_{scale} = \frac{1 + LSWI}{1 + LSWI_{max}} \quad (3)$$

LSWI is land surface water index, and is a proxy for water stress.

$$L_{scale} = \frac{PAR}{1 + PAR/PAR_0} \quad (4)$$

PAR is equal to 2*shortwave fluxes, and shortwave fluxes account for 50% of the incident solar radiation. PAR₀ is the half saturation value, determined empirically for different vegetation types. Total ecosystem respiration can be parameterized as:

$$R = \alpha \cdot (\max(T, T_{low}) - 273) + \beta \quad (5)$$

This relationship assumes that the rate of soil + plant respiration is linear with temperature, and that even when temperatures fall below T_{low} , soil respiration occurs. In the equation for respiration, α and β are determined empirically. For a typical midlatitude forest, empirical parameters are given in Table 1.

Table 1: Parameters for Vegetation Model

λ	0.1	$[\mu\text{mol CO}_2 \text{ s}^{-1} \text{ m}^2]$
EVI	0.3	0.3
T_{min}	0	[deg C]
T_{max}	40	[deg C]
T_{opt}	20	[deg C]
T_{low}	5	[deg C]
LSWI	0.7	[.]
$LSWI_{max}$	1	[.]
α	0.265	$[\mu\text{mol CO}_2 \text{ s}^{-1} \text{ m}^2 \text{ K}^{-2}]$
β	0.25	$[\mu\text{mol CO}_2 \text{ s}^{-1} \text{ m}^2]$
PAR_0	570	$[\text{W m}^2]$

You can compute daily insolation at 45 deg N using `dailyinsolation.m`, and use 2004-2007 mean temperatures for a forest in northern Wisconsin from the file `DailyMeanTemp.txt`. Plot GPP and R for one year, and explain the seasonality in these quantities. How long is the growing season (ie, net flux of CO_2 from the atmosphere into the biosphere)? What is the net flux of CO_2 into the biosphere during the growing season?

(3) How does GPP vary as a function of temperature, water, and light? Make plots showing GPP as a function of each of these variables and explain the response of GPP to each.

(4) Now, plot the seasonal cycle in atmospheric CO_2 , assuming respiration and photosynthesis are the only fluxes. What is the amplitude? Explain what might cause deviations between the modeled amplitude and an observed amplitude of 12 ppm.

(5) What happens if the temperatures increase by 2 degrees? How does this change the balance between GPP and R and the growing season length/onset? What happens if LWSI decreases by 5%? What happens if temperatures increase by 2 K and LWSI decreases by 5%? What does this simple model suggest about how the biosphere might respond to climate change?

(6) What are some of the problems you anticipate in modeling GPP and respiration as we have above? What assumptions have we made that do not capture feedbacks that occur in the Earth system?