Problem set 5

Ge/Ay 133

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1 Linblad Resonances

In the limit of quasi-circular orbits, small perturbations that lead to radial oscillations create what are known as epicycles. The epicyclic frequency is typically denoted as κ. For sufficiently small perturbations, the radial excursions can be treated as a harmonic oscillator problem, leading to the relationskip $\kappa^2 = r^{-3} d/dr[r^4 \Omega^2]$, where Ω is the orbital angular velocity.
(a) From this, verify the assertion that the epicyclic and orbital frequencies are the same for a Keplerian disk.
(b) What are known as the Linblad resonances occur at those locations where the driving force from a protoplanet is at the natural frequency for epicyclic oscillations. Mathematically, the following conditions must be met:

$$m[\Omega(r) - \Omega_p] = \pm\kappa(r) \quad ,$$

where m is an integer and Ω_p is evaluated at the planet's orbital radius. From this equation and your answer to part (a), derive an equation for the location of the Linblad resonances in a Keplerian disk with an embedded protoplanet.
(c) For a sufficiently massive object, protoplanet-disk interactions at the Linblad resonances will open up a gap in the disk. Before this happens, torques on the protoplanet can cause it to move. Verify numerically the estimates for migration timescales before a gap is opened using eq. (7.10) on p. 246 of the Armitage text for the one Earth mass and one Jupiter mass cases considered.

2 Gaps and migration

(a) When sufficiently large planets open gaps in disks, their radial motions becomes tied to the evolution of the gas in the disk. Thus, if the disk gas is evolving on the viscous timescale (remember Problem Set 3?), the planet migrates on this viscous timescale. Based on the viscous disk evolution timescale, write an approximate expression for the speed of the migration for a giant planet which has formed a gap.
(b) Assume that every planetary system forms planets at 5 AU and then these planets migrate inwards on the viscous timescale. Then, at some random time, the gas in the disk instantly disappears (halting migration). If this happens in a huge set of planetary systems, what is the DISTRIBUTION of semi-major axes might you expect to observe in planetary systems? (You can try your own simulation, or else create a sketch of what you think it will look like).
(c) Compare this predicted distribution to that of extrasolar planets (http://www.exoplanets.eu, in the interactive catalog portion of the website). Be sure to consider the (severe) biases in the discoveries of extrasolar planets.
(d) In what ways is the process suggested in part (a) realistic? In what ways is it not?