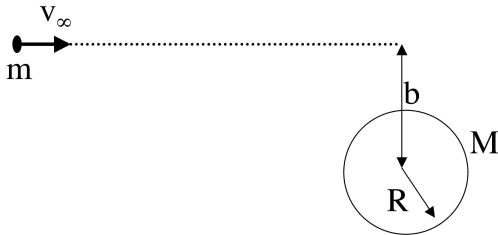


Problem set 4

Gravitational Focusing

One of the most important concepts in pairwise accretion is the “gravitational focusing” caused by the deflection of a small body by a massive body. This deflection works to increase the cross-section for collision of the two bodies. Your goal is to reproduce a calculation first done by Safronov. Try this with no books, no notes, no discussion with anyone. Really. Consider a test particle approaching a body of mass M and radius R . The impact parameter is b and the velocity at infinite distance is v_∞ (i.e. in the absence of gravity, the two bodies would approach within a distance of b of each other and with relative velocity v_∞). In the absence of gravity, the cross-section for collision would be πR^2 . We would like to know by how much this cross-section increases due to the effects of gravity.



- What is the escape velocity from the large body? (you'll need this for later).
- Draw a diagram of a *grazing* collision between the two bodies. (A grazing collision means that if the point mass had just slightly more energy it would not collide with the massive body) Pay particular attention to where the two bodies touch.
- For the case you just drew, use the principle of conservation of angular momentum to write an equation relating b , v_∞ , R , and v_{imp} , the velocity of the object at impact. (Hint: Write down the total angular momentum in the 'before' picture, shown above, and the total angular momentum at the moment of impact, and set these equal)
- Write an energy conservation equation relating v_∞ , v_{imp} , R , and M .
- What is the maximum value for b for which an impact will occur? Write your answer in the form of $b^2 = R^2(1 + f)$ where f is called the focusing or Safronov parameter, which should be expressed in terms of the escape velocity.
- (you can start collaborating again at this point) How large does an object need to be before gravitational focusing significantly increases the impact (and thus accretion) cross-section. The answer to this question is not obvious without some additional information that you don't necessarily possess, so do your best and make reasonable estimates where necessary. Justify your answer carefully.
- An important concept that we will discuss next week is that of *dynamical friction*, in which large bodies are slowed by the presence of a swarm of smaller bodies. This process leads to an equipartition of energy between small and large bodies such that $mv^2 = MV^2$ where the lowercase values refer to small bodies and the uppercase to large bodies. Explain how this dynamical friction could greatly speed up the growth of large bodies.