Ge/Ay133 – Problem Set #2 Due October 14^{th}, 2009

The goal of this problem set is to see if we can understand why extrasolar planets close to their parent star have circular orbits. Assume below that the mass of the star is M_s , the mass of the planet is m_p , the initial semimajor axis of the planet is a and the initial eccentricity is e. Be ruthless in making any approximations that help **except** we unfortunately can't assume that $e << 1$.

(a) First, we need to make sure we understand the orbit. If the planet has semi-major axis a and eccentricity e, what are the peristron and apistron distances (closest to the star and most distant from the star, respectively). If you know the answer, feel free to just write it down. If you dont know the answer, you will likely need to go back to your high school geometry and remember that the star is at one focus of the ellipse.

(b) Now a quick reminder of how the energy and angular momentum of an orbit scales with a and e. Assume that the planet moves with velocity v at peristron. (i) Conserving angular momentum, what is its velocity at apistron? (ii) What is its total kinetic plus potential energy at peristron? At apistron? [be sure to think twice about the sign of the potential energy] (iii) Setting these energies equal (which they had better be...) solve for v^2 and check to make sure that you have the right answer by comparing to your expectation for a circular orbit (if the expression looks ugly keep trying to simplify...). (iv) What is the total energy of the orbit as a function of a and e (and M_s and G, of course). (v) What is the total angular momentum of the orbit as a function of a and e (and the constants)?

(c) Now we need to understand tides. If the planet is a distance d from the star, what is the tidal force on the planet? Specifically, assume that the planet has (small) radius r and calculate the difference in force between the part of the planet closest to the star and the part of the planet furthest from the star. This difference is the force trying to pull the planet apart.

(d) Why don't these close planets get ripped apart by these tidal forces? The answer must be self-gravity. Assuming solar and Jupiter masses and sizes for the star and planet, respectively, how close can the planet get before it will be pulled apart by tides? Compare this distance to those at which (exo)planets are observed.

(e) Assume that, like a spring, the size of the tidal bulge is proportional to the tidal force with a "spring" constant of proportionality equal to k such that $F = kb$ where b is the elongation and F is the force. (this assumption is not exactly true, since we should really take into account the self-gravity as above, but that gets more complicated). Estimating k is hard. But lets try. To order of magnitude, estimate the proportionality constant for the oceanic tides raised on the earth by the moon using this simple formula.

(f) What is the difference in size of the tidal bulge between the peristron and apistron of the planet? Call this ϵ_b. In this case only, it is OK to assume $e << 1$. Why? Because it is too ugly otherwise. Also because we have such high powers of e.

(g) Every orbit, the planet is stretched and compressed by these differing tides. In an ideal system (like a perfect spring) such stretching and compressing would take no energy, but in real life compressing and relaxing release heat, which requires energy. Assuming that the energy loss per orbit is proportional to ϵ_b, write an expression for the amount of energy lost per orbit as a function of a and e. Change this expression to dE/dt, where we're now looking for the change in energy per second (averaged over an orbit).

(h) Ignoring rotation of the star and planet, the only source for the energy above is the orbit. Write expressions for da/dt and de/dt given this dE/dt (it helps if you rewrite e in terms of E).

(i) What happens to the orbit of the planet as energy is lost to the tidal flexing? What is the eventual fate of the planet? Sketch a plot being as quantitative as possible.

(j) What happens if we change the values of k or E? What do you think these changes physically correspond to?

(k) Whew. In words, rather than equations, explain what all of this means and how it relates to the orbital parameters seen for extrasolar planets. What can be learned from the location of the tidal circularization radius in extrasolar planets?