GE/AY 133 Fall 2009, Problem Set 1 Solutions

1. *Consider a star of mass* M_* *around which orbits a planet of mass* M_p *at an orbital distance* r. *For a star some* R *parsecs distant, use Kepler's laws to derive the maximum radial velocity and astrometric wobble of the star (that is, assume the inclination angle is 90 degrees). For a solar mass star 10 pc away, what are the numerical values of the radial velocity and wobble for a Jupiter mass planet at 5 AU? A Venus mass planet at 0.5 AU?*

The maximum radial velocity of the star due to the planet is independent of R. For a circular orbit with inclination 90º, it is equal to the orbital velocity of the star about the center-of-mass of the planet-star system. As we discussed in class, lower inclinations produce radial velocities that are lower by a factor of sin(*i*), but the maximum astrometric wobble remains the same.

For a large mass ratio between the star and the planet, the star's orbital velocity is:

$$V_* = V_p M_p / M_*$$

V_p, the orbital velocity of the planet, is derived using Kepler's 3rd law:

$$V_p = 2\pi r/P; \;\; P = (4\pi^2 r^3/GM_*)^{1/2}; \; V_p = (GM_*/r)^{1/2}$$

and $V_* = (GM_*/r)^{1/2} M_p/M_*$

Again for a large mass ratio, the orbital radius of the star about the center-of-mass is:

$$r_* = rM_p/M_*$$

The astrometric wobble of the star will have a maximum value (relative to the center-of-mass) of the orbital radius divided by R:

$$\Theta = r_*/R = rM_p/RM_*$$

For $M_* = 1\ M_{Sun}$, $M_p = 1\ M_J$, r = 5 AU, R = 10 pc:

V_* = 12.7 m/s; Θ = 2.31e-9 radians = 480 microarcseconds.

For $M_* = 1\ M_{Sun}$, $M_p = 1\ M_V$, r = 0.5 AU, R = 10 pc:

V_* = 0.103 m/s; Θ = 5.92e-13 radians = 0.122 microarcseconds.

These values illustrate the difference between radial velocity and astrometric detection: both favor high-mass planets, but astrometry is far more effective for planets at large r (although it takes much longer to collect data spanning a significant fraction of an orbit).

2. For this same system, a solar mass star at an inclination angle of exactly 90 degrees with one planet, how does the transit depth scale with orbital distance from the star? For a Jupiter mass planet at 5 AU how deep is the transit, assuming no limb darkening for the star? For an Earth at 1 AU?

The transit depth is independent of orbital distance from the star, since we are so far away from the system that the angular size of the planet does not change significantly. The orbital distance determines the transit times and the range of inclinations over which the planet will transit the star.

If the star has no limb darkening, the transit depth will be the amount of light blocked by the planet divided by the total light from the star:

$d_T = (\pi R_p^2)/ (\pi R_*^2) = (R_p/R_*)^2$ = 0.0106 for Jupiter orbiting the Sun
= 8.41e-5 for Earth orbiting the Sun

Calculate the following transit times for a Jupiter mass planet at 5 AU:

The full transit from first to last contact (t_T):

$t_T = (2R_*+2R_p)/V_p = (2R_*+2R_p)(r/GM_*)^{1/2}$ = 115000 s for Jupiter

The transit time over which the planet fully occults the star (t_F):

$t_F = (2R_*-2R_p)/V_p = (2R_*-2R_p)(r/GM_*)^{1/2}$ = 93700 s for Jupiter

These equations assume that the relative velocity of the star and the planet is equal to the planet's velocity, and that the angular velocity of the planet is constant, so they are accurate to a fractional precision of ~ 0.001 for Jupiter and ~0.01 for a hot Jupiter at 0.1 AU.

Both transit times must include the diameter of the star ($2R_*$). The time from first to last contact also includes an additional width of R on either side of the star – the center of mass of the planet is this far from the limb of the star at those times. Similarly, the fully-occulted time requires that the entire planet be over the stellar disc.

From these, how long are the ingress/egress times? This tells you about the cadence you'd like to take data with.

Ingress and egress take $2R_*/V_p$ = 10700 s for Jupiter.

Data points should be taken with spacing considerably less than the ingress/egress time. E.g. for a hot Jupiter at 0.05 AU, the full transit would last 11500 s, ingress/egress 1070 s, and data points should be spaced no more than ~300 s apart.

Can the relative values of t_T *and* t_F *tell you at what ``latitude'' the planet crosses the star?*

By "latitude" we denote the position of the transit on the stellar disc – if it spans a full diameter, the latitude is 0º, if it just grazes the limb the latitude is 90º. A non-zero latitude is due to the inclination of the system being slightly less than 90º.

t_T and t_F (and the ingress/egress times) do change with the latitude, so the answer to this question is 'yes'. The total duration of the transit decreases as latitude goes up, since the path across the star is shorter, and the difference between t_T and t_F increases, since the time from first contact to full occultation is longer when the limb of the star is not normal to the motion of the planet. In the limit where the planet just grazes the top of the stellar disc, t_F is equal to zero.