Ge 102 Winter 2008
Due to Francisco on Wednesday, Feb 20, 2008

A. Show that a uniform density sphere has moment of inertia, $I = FMa^2$, where $F = 0.4$, M is the mass and a is the radius of the sphere. For the Earth, $F = 0.3307$ - what does this imply? Through trial and error (or any other approach), can you come up with a 2 layer Earth that satisfies the observed values of F and the mean density? Begin by assuming that the outer layer has a density of 3300 kg/m^3.

B. The gravitational potential at a radius, R_{obs}, relative to values at a reference radius, R_s, can be expressed in terms of spherical harmonics:

$$U(\theta,\phi) = -\frac{1}{R_s}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{R_s}{R_{obs}}\right)^{l+1}[C_{lm}\cos(m\phi) + S_{lm}\sin(m\phi)]P_{lm}(\cos(\theta)) \tag{1}$$

for $R_{obs} \geq R_s$. Here, C_{lm} and S_{lm} are harmonic coefficients of degree l and order m, P_{lm} are fully normalized Legendre polynomials, and θ and ϕ are, respectively, colatitude and longitude. The coefficients for the potential U_{lm} outside of a single spherical harmonic mass sheet (see *Sleep and Fujita* [1997], Principles of Geophysics, p490) is:

$$U_{lm} = \Gamma_{lm}\frac{4\pi G R_s}{2l+1}\left(\frac{R_s}{R_{obs}}\right)^{l+1} \tag{2}$$

where R_s is the mean radius of the mass sheet, $\Gamma_{lm}(= H_{lm}\rho_c)$, are the coefficients of the mass sheet, H_{lm} are the harmonic coefficients of topography, and ρ_c is the crustal density. Show that the coefficients for the geoid anomaly, N_{lm}, $(= U_{lm}/g_o)$ at the surface of a planet, where all topography is Airy compensated at a depth D, can be written as:

$$N_{lm} = H_{lm}\frac{3\rho_c}{\overline{\rho}}\frac{1}{2l+1}\left(1 - \left(\frac{R_0 - D}{R_0}\right)^l\right) \tag{3}$$

where $\overline{\rho}$ is the average density of the planet, R_0 is the radius of the Earth, and g_0 is the nominal value of gravity at the surface fo the Earth. Note that because of the convergence of the verticals in a spherical planet,

$$\Gamma_{lm}^{bottom} = -\left(\frac{R_{top}}{R_{bottom}}\right)^2\Gamma_{lm}^{top} \tag{4}$$

(you do not need to prove this last point).